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INFLUENCE OF LONGITUDINAL COMPRESSION WAVE AND SHEAR WAVE IN THE LONG PIPE WITH A LIQUID

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Abstract:

The article covers the problem of dynamic theory of linear elasticity when seismic wave incidences perpendicular to the axis of long pipe laid in a high embankment and filled with ideal compressible fluid. The design scheme is shown in Fig. 1.
Equation of motion in vector form for isotropic body known from the dynamic theory of elasticity has been obtained (1)

Keywords: Long pipe, linear elasticity, seismic wave

INTRODUCTION

The article covers the development of methodology for solving the problem of impact of the propagation of dynamic theory of linear elasticity when , incidences perpendicular to the axis of long pipe, laid in a high embankment and filled with an ideal compressible fluid. The problem of dynamic theory of linear elasticity when a seismic wave incidence perpendicular to the axis of a long pipe laid in a high embankment and filled with an ideal compressible fluid has been considered. The design diagram is shown in Figure 1.

The equation of motion in vector form for an isotropic body, known from the dynamic theory of elasticity, has the form: [1, 2]

$$(\vec{\lambda}+\vec{\mu})gradd\ iv\vec{u} - rot\,rot\vec{u} + \vec{f} = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

where$\rho$is the density of the medium, and all other designations have the same meaning as in the equation of the static theory of elasticity [8]. Standard transformation of the equation will be made as follows. The displacement vector is represented in the form: [4, 5]

$$\vec{u} = grad\varphi + rot(\vec{\psi}). \quad (2)$$

Substituting (3.1.2) into (3.1.1) and taken into consideration that the motion of the particle has a steady character, and also neglecting the mass forces, f=0,since in accordance with the principle of superposition, they can be taken into consideration separately when solving a static problem, in the case of plane deformation, we obtain the following system of Helmholtz wave equations for potentials: [6, 7]

$$\Delta\varphi + \alpha^2\varphi = 0;$$
$$\Delta\psi + \beta^2\psi = 0 \quad , \quad (3)$$

where$\alpha$and$\beta$are wave numbers

$\alpha^2=\omega^2\rho/(\lambda+2\mu)$,
$\beta^2=\omega^2\rho/\mu$. (4)

In a polar coordinate system, the Helmholtz equation can be written in the form:[6, 7]

$$V_{rr}+r^{-1}V_r+r^{-2}V_{\theta\theta}+k^2V=0 \quad , \quad (5)$$

where $V=(\varphi,\Psi)$; $k=\alpha;\beta$.

The solution to equation (5) is sought in the form of a series:

$$V = \sum_{n=0}^{0}\left[V_n^{\ a}(r)\cos n\theta + V_n^{\ b}(r)\sin n\theta\right]e^{-i\omega t}. \quad (6)$$

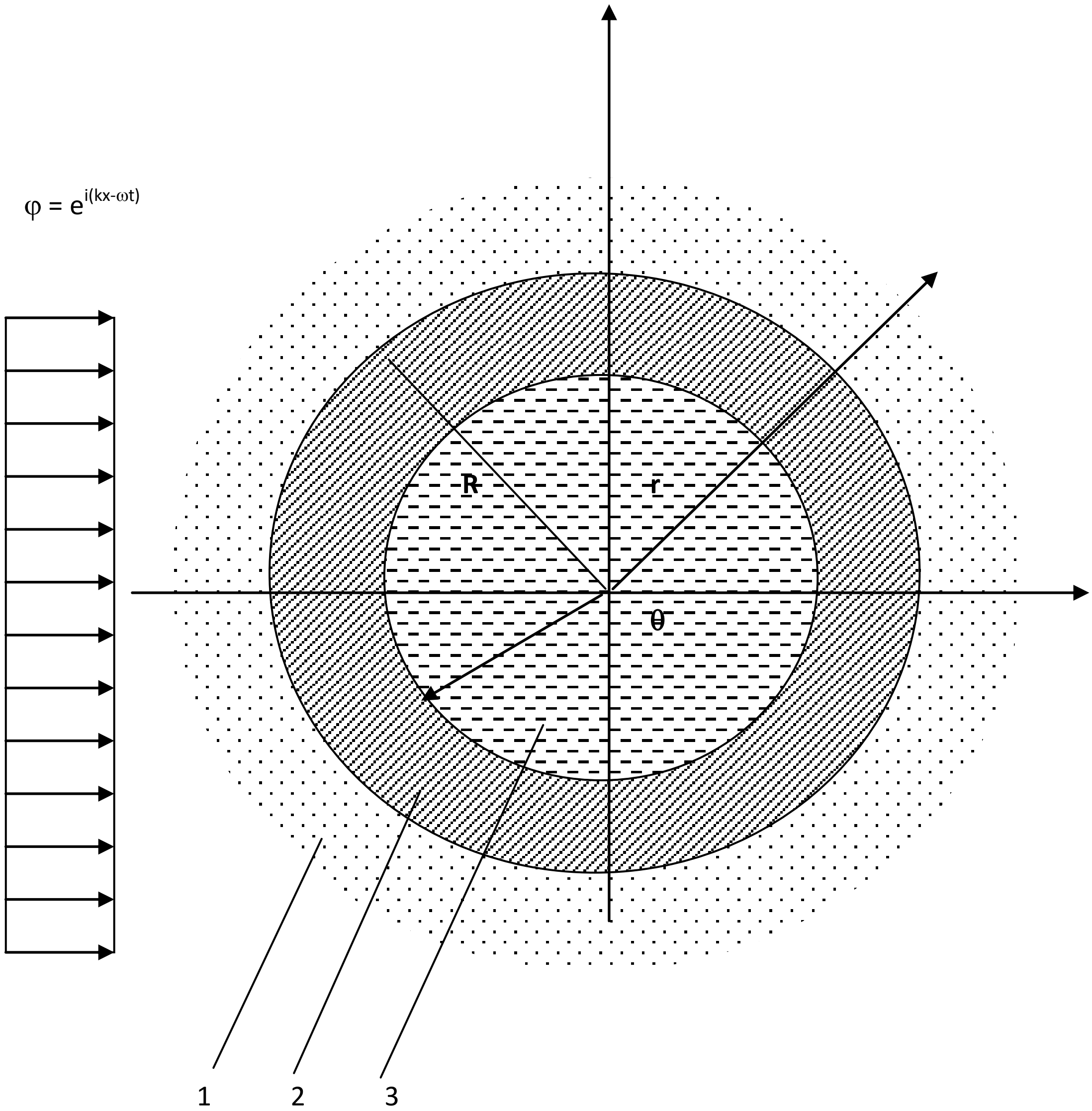

Fig. 1 Design diagram: 1-soil; 2-pipe; 3-liquid.

Substituting (6) into (5) and equating the coefficients at the corresponding harmonics, we obtain the ordinary differential Bessel equation [5, 6]

$r^2V''_n+rV'_n+(k^2r^2-n^2)V_n=0.$ (7)

which has a particular solution in the form of a cylindrical function $Z_n(kr)$. Then the final solution to system (1) will be written in the form:

$$U_r=\sum_{n=0}^{\infty}A_nZ_n(\alpha r)\cos n\theta e^{-i\omega t},$$

$$U_\theta=\sum_{n=0}^{\infty}B_nZ_n(\beta r)\sin n\theta e^{-i\omega t}. \quad (8)$$

Now we put solutions (6) in($r=\infty$) of the Sommerfeld radiation condition [10, 12], which has the form

$$\varphi,\psi=0\ (1/\sqrt{r}),\ \varphi_r\mp i\alpha\varphi=0\,(1/\sqrt{r})$$

$$\psi_r\mp i\beta\psi=0\ (1/\sqrt{r}). \quad , \quad (9)$$

At r=R,condition for perfect contact of the soil with the pipe

$$U_{r1}\Big|_{r=R_0}=U_{r2}\Big|_{r=R_0}\ ;\quad U_{\theta 1}\Big|_{r=R_0}=U_{\theta 2}\Big|_{r=R_0}\ ;$$

$$\sigma_{rr1}\Big|_{r=R_0}=\sigma_{rr2}\Big|_{r=R_0}\ ;\quad \sigma_{r\theta 1}\Big|_{r=R_0}=\sigma_{r\theta 2}\Big|_{r=R_0}\ . \qquad (10)$$

At $r=R_0$,condition for perfect contact of the water with the pipe

$$\frac{\partial U_{r2}}{\partial t}\Big|_{r=R_0}=\frac{\partial U_{r3}}{\partial t}\Big|_{r=R_0}\ ;\sigma_{rr2}\Big|_{r=R_0}=\sigma_{rr3}\Big|_{r=R_0}\ ;$$

$$\sigma_{r\theta 2}=\Big|_{r=R_0}=0\ . \qquad (11)$$

where indices 1, 2 and 3 correspond to soil, pipe and liquid.

Note that in the case of sliding soil contact over the pipe surface, the last equation in (10) takes the form

$\sigma_{r\theta 1}=0$ (12.)

In addition, in the absence of liquid in the pipe, the first equation in (2) will be written in the form

$\sigma_{rr2}=0$ (13)

Moreover, the third equation will disappear.

Taking into account the obtained relations, we derive the solution of the boundary value problem for the case of a compression wave falling on an underground pipe. The wave potential of such a wave has the form [13, 14]

$$\varphi_1^{(i)}=Ae^{i(\alpha_1 x-\omega t)}\ , \qquad (14)$$

where A is the amplitude of the incident P-wave. In order to represent (14) in the form (8), we will write (14) in polar coordinates, and then expand in a Fourier series (complex Form) and use the integral definition of the Bessel function [5]

$$2\pi i^n I_n(Z)=\int_0^{2\pi} e^{iZ\cos\theta}\sin n\theta d\theta \qquad (15)$$

Then we have

$$\varphi_1^{(i)}=A\sum_{n=0}^{\infty}\in_n i^n I_n(\alpha_2 r)\cos n\theta e^{-i\omega t}\ , \qquad (16)$$

where

$$\in_n=\begin{cases}1, & \mathrm{n}=0\\ 2, & \mathrm{n}\geq 1\end{cases}\ , \qquad (16a)$$

I_n iscylindrical Bessel function of the first kind [98].

The potentials of waves reflected from the pipe into the ground then has the form (8) and at the same time satisfies the radiation conditions (9), therefore, according to [14], are written in the form:

$$\varphi_1^{(r)}=\sum_{n=0}^{\infty}A_n H_n^{(1)}(\alpha_1 r)\cos n\theta e^{-i\omega t}\ , \qquad (17)$$

$$\psi_1^r=\sum_{n=0}^{\infty}B_n H_n^{(1)}(\beta_1 r)\sin n\theta e^{-i\omega t}$$

where $H_n^{(1)}$ isthe cylindrical Hankel function of the first kind [9]. The total potentials in the soil are equal to:

$\varphi_1=\varphi_1^{(i)}+\varphi_1^{(r)}$; $\psi_1=\psi_1^r$ (17a)

The waves refracted in the pipe at the beginning propagate towards the center of the pipe, and then, being reflected, go in the opposite direction. Therefore, they must satisfy both the conditions of radiation and absorption [4]:

$$\varphi_2^{(r)}=\sum_{n=0}^{\infty}\left[C_n H_n^{(1)}(\alpha_2 r)+D_n H_n^{(2)}(\alpha_2 r)\right]cosn\theta e^{-i\omega t}$$

$$\psi_2^{(r)}=\sum_{n=0}^{\infty}\left[C_n H_n^{(1)}(\beta_2 r)+F_n H_n^{(2)}(\beta_2 r)\right]\sin n\theta e^{-i\omega t}\ , \qquad (18)$$

where $H_n^{(2)}(\alpha)$ isthe cylindrical Hankel function of the second kind of the n[th]order. The velocity potential in a compressible fluid has the form:

$$\varphi_3 = \sum_{n=0}^{\infty}\left[G_n I_n(\alpha_3 r)\right]\cos n\theta e^{-i\omega t} \qquad (19)$$

The components with the index "3" (liquid) were obtained according to [9, 11] using the linearized Cauchy-Lagrange integral for the hydrodynamic pressure of an ideal compressible fluid.

Substituting (16), (9) in (10), we obtain the final solution of the problem posed for the case of a P-wave falling on an underground pipe:

$$U_{r1} = \sum_{n=0}^{\infty}\begin{bmatrix} AE_n i^n I_n^1(\alpha_1 r)\alpha_1 + A_n H_n^{(1)`}(\alpha_1 r)\alpha_1 + \\ + nr^{-1} B_n H_n^{(1)}(\beta_i r) \end{bmatrix}\cos n\theta e^{-i\omega t} \qquad (20)$$

$$U_{r2} = \sum_{n=0}^{\infty}\begin{bmatrix} C_n H_n^{(1)`}(\alpha_1 r)\alpha_2 + D_n H_n^{(2)`}(\alpha_1 r)\alpha_2 + \\ + nr^{-1}(E_n H_n^{(1)}(\beta_n r) + F_n H_n^{(2)}(\beta_n r)) \end{bmatrix}\cos n\theta e^{-i\omega t}$$

$$U_{r3} = -(i\omega)^{-1}\sum_{n=0}^{\infty}\alpha_3 G_n I_n`(\alpha_3 r)\cos n\theta e^{-i\omega t} \qquad (21)$$

$$U_{\theta 1} = \sum_{n=0}^{\infty}\begin{bmatrix} -nr^{-1}(AE_n i^n J_n(\alpha_1 r) + A_n H_n^{(1)}(\alpha_1 r)) - \\ - B_n H_n^{(1)`}(\beta_1 r)\beta_1 \end{bmatrix}\sin n\theta e^{-i\omega t}$$

$$U_\theta = \sum_{n=0}^{\infty}\left[-nr^{-1}(C_n H_n^{(1)}(\alpha_2 r) + D_n H_n^{(2)}(\alpha_2 r) - \right.$$
$$\left. - E_n H_n^{(1)`}(\beta_2 r)\beta_2 - F_n H_n^{(2)}(\beta_2 r)\beta_2\right]\sin n\theta e^{-i\omega t},$$

$$\sigma_{rr1} = -\sigma_{\theta\theta 1} = \sum_{n=0}^{\infty}\left[-\alpha_1\alpha_1^2(AE_n i^n I_n(\alpha_1 r) + A_n H_n^{(1)}(\alpha_1 r) - \right. \qquad (22.)$$
$$- 2\mu_1 r^{-1}(AE_n i^n I_n^1(\alpha_1 r)\alpha_1 + A_n H_n^{(1)}(\alpha_1 r)\alpha_1) +$$
$$'+2\mu_1 r^{-2} n^2(AE_n i^n I_n^1(\alpha_1 r) + A_n H_n^{(1)}(\alpha_1 r) -$$
$$\left. - 2\mu_1 r^{-2} n B_n H_n^{(1)}(\beta_1 r) + 2\mu_1 r^{-1} B_n H_n^{(1)}(\beta_1 r)\beta_1\right]\cos n\theta e^{-i\omega t}$$

$$\sigma_{rr2} = \sum_{n=0}^{\infty}\left[-d_2\alpha_2^2(C_n H_n^{(1)}(\alpha_1 r) + DH_n^{(2)}(\alpha_1 r)) - \right.$$
$$- 2\mu_2 r^{-1}(C_n H_n^{(1)`}(\alpha_2 r)\alpha_2 + 2\mu_2 r^{-2} n^2(C_n H_n^{(1)`}(\alpha_2 r) +$$
$$+ D_n H_n^{(1)`}(\alpha_2 r) - 2\mu_2 r^{-2} n(E_n H_n^{(1)}(\beta_2 r) + F_n H_n^{(2)}(\beta_2 r)) +$$
$$\left. + 2\mu_2 r^{-1} n(E_n H_n^{(1)`}(\beta_2 r)\beta_2 + F_n H_n^{(2)`}(\beta_2 r)\beta_2)\right]\cos n\theta e^{-i\omega t},$$

(23)

$$\sigma_{r\theta 1} = \sum_{n=0}^{\infty}\left[\mu_1\beta_1^2 B_n H_n^{(1)`}(\beta_1 r) + 2\mu_1 r^{-1} B_n H_n^{(1)`}(\beta_1 r)\beta_1 - \right.$$
$$- 2\mu_1 r^{-2} n(AE_n i^n I_n`(\alpha_1 r) + A_n H_n^{(1)`}(\alpha_1 r)) -$$
$$\left. 2\mu_1 r^{-1} n(AE_n i^n I_n`(\alpha_1 r)\alpha_1 + A_n H_n^{(1)`}(\alpha_1 r)\alpha_1)\right]\cos n\theta e^{-i\omega t}$$

$$\sigma_{r\theta 2} = \sum_{n=0}^{\infty}\left[\mu_2\beta_2^2(E_n H_n^{(1)}(\beta_2 r) + F_n H_n^{(2)}(\beta_2 r)) + 2\mu_2 r^{-1}(E_n H_n^{(1)`}(\beta_2 r)\beta_2 + \right.$$
$$+ F_n H_n^{(2)}(\beta_2 r)\beta_2) - 2\mu_2 r^{-1}(E_n H_n^{(1)`}(\beta_2 r) + F_n H_n^{(2)}(\beta_2 r)) + \qquad (24)$$
$$+ 2\mu_2 r^{-1}(C_n H_n^{(1)`}(\alpha_2 r) + D_n H_n^{(2)}(\alpha_2 r)) + 2\mu_2 r^{-1}(C_n H_n^{(1)`}(\alpha_2 r)\alpha_2 +$$
$$\left. + D_n H_n^{(2)`}(\alpha_2 r)\alpha_2\right]\sin n\theta e^{-i\omega t}$$

The known coefficients A_n, B_n, C_n, D_n, E_n, F_n, G_n are determined from the system of linear equations of the seventh order, which is obtained by substituting (12), (17) in (10) and (1.1) and has the form (matrix notation):
[C]{q}={P} (25),

where, [C] is square matrix (7*7); {q} is vector column of unknown values; {P} is vector column of external loads,

for example, some elements of the matrix [C] is shown below

$$C_{11} = \alpha_1 H_n^{(1)`}(\alpha_1 R)$$
$$C_{12} = nR^{-1} H_n^{(1)}(\beta_1 R)$$
$$C_{13} = -\alpha_2 H_n^{(1)`}(\alpha_2 R)$$
$$C_{14} = -\alpha_2 H_n^{(2)`}(\alpha_2 R)$$
$$C_{15} = -nR^{-1} H_n^{(1)}(\beta_2 R)$$
$$C_{16} = -nR^{-1} H_n^{(2)}(\beta_2 R)$$
$$a_1 = -AE_n i^n \alpha_1 I_n^1(\alpha_1 R)$$

Note that in the case of soil slippage along the pipe surface, according to (12), in (18) should set [17, 18]

$$C_{2i} = a_2 = 0; \quad i=1.6 \quad (26)$$
$$C_{43} = C_{44} = C_{45} = C_{46} = 0.$$

In addition, in the absence of liquid in the pipe:

$$\sigma_{rr} = 0. \quad (27)$$

Now let us consider the case of a plane SV-wave incidence on an underground pipe with liquid perpendicular to the pipe axis. The wave potential of such a wave, by analogy with (16), has the form:

$$\psi_1^{(i)} = B\sum_{n=0}^{\infty} E_n i^n I_n(\beta_n r)\sin n\theta e^{-i\omega t},$$

whereB is the amplitude of potential of the incident SV-wave. The form of the remaining potentials (17), (19) remains unchanged, and the total potentials in the soil have the form [9.12]

$$\varphi_1 = \varphi_1^{(i)}, \psi_1 = \psi_1^{(i)} + \psi_1^{(r)}.$$

Disposition and stress in this case are taken in the form:

$$U_{r1} = Br^{-1}\sum_{n=0}^{\infty} E_n i^n I_n(\beta_1 r)\cos n\theta e^{-i\omega t};$$

$$\sigma_{rr1} = 2\beta_1\mu_1 n r^{-1}\sum_{n=0}^{\infty} E_n i^n\left[-r^{-1}I_n(\beta_1 r) + I`_n(\beta_1 r)\beta_1\right]\cos n\theta e^{-i\omega t};$$

$$U_\theta = -B\sum_{n=0}^{\infty} E_n i^n I_n`(\beta_1 r)\beta_1 \sin n\theta e^{-i\omega t},$$

$$\sigma_{r\theta 1} B\sum_{n=0}^{\infty} E_n i^n \mu_1\left[\beta_1^2 I_n(\beta_1 r) - 2n^2 r^{-2} I_n(\beta_1 r) + \right.$$
$$\left. + 2r^{-1} I_n`(\beta_1 r)\beta_1\right]\sin n\theta e^{-i\omega t}.$$

Other components $U_{r2.}$ $U_{\theta 2.}$ $U_{r3.}$ $\sigma_{rr2.}\sigma_{\theta\theta 2.}$ $\sigma_{rr3.}\sigma_{\theta\theta 3.}\sigma_{r\theta 2}$ are determined, respectively, by formulas (17). To obtain undefined coefficients, system (18) can be used, in which, in comparison with the case for the wave, only equal parts change. For example, the free term is compared with the formula (19) is written in the form: [15, 16]

$$a_1 = -nR^{-1}E_n i^n I_n(\beta_1 R)B. \quad (28)$$

In the case of slippage of the soil along the surface of the pipe or the absence of liquid filling it, formulas (21) are true, respectively.

In addition to the impact on the underground pipe of waves directed perpendicular to its axis, waves directed along the pipe axis, in particular, shear wave SH, are also of considerable interest. [20, 22]

From the point of view of design practice, it is necessary to know at what distance the pipes can be laid so that the dangerous phenomenon of resonance does not arise.

The answer to this question is given by the ratio. Let us analyze this ratio for the case of the impact of P- and SV- seismic waves on underground pipeline. Table 1 shows the dependence of the maximum clear distance between the pipe centers d_{max}, at which no resonance on the angle of incidence of seismic waves γoccurs.

Table 1.
Dependence of distance D_{max}on the angle of incidence φ.

φ, degree	0	30	45	60	70	80	90
D_{max}, m	5.0	5.36	5.86	6.66	7.45	8.52	10.0

From Table 1 it follows that the smaller the angle of incidence of the seismic wave on the pipeline, the closer to lay the pipes to each other is necessary. Therefore, the appearance of resonance in multi-stranded pipes can be avoided by choosing an appropriate distance between them and, therefore, to ensure the seismic resistance of the pipeline. Influence of the type of seismic impact (P-, SV-or SH-wave). Table 1 shows the values of η_{max} of the maximum radial soil pressure on pipes in the case of a P- and SV- seismic wave fall at different distances d in the clear between the pipes. In this case, it was assumed that β_r=2.

Analysis of the data in Table 2 shows that at d/D<4.0, values of the coefficient $\eta_{max}$for the P- and SV-waves is, as it were, in antiphase, i.e. at l/D = 1.0, the maximum seismic effect of the P-wave is 27% higher than that of the SV-wave, at d/D = 2.0 it is 7% lower, and at d/D = 4.0 it is again higher, but already by only 1 %.

Table 2.
The value of the coefficient η_{max}under seismic influences in the form of P- and SV- waves at different distances d between the pipes

d/D	η_{max}	
	P-wave	SV-wave
1.0	1.76	1.29
2.0	1.61	1.72
4.0	1.60	1.51

Moreover, with an increase in the distance between the pipes, the difference in these effects decreases, and at d/D = 4.0 practically disappears at all. In addition, we note that under the action of the SV - wave, the values of η_{max}at various distances between the pipes have a 2.5 times greater spread (up to 25%) than under the action of the P - wave (up to 10%). Thus, the phenomenon of "local resonance" is more revealed for the seismic impact in form of SV-wave.[21,23.]

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