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THEME. ON THE QUALITATIVE PICTURE OF THE TRAJECTORY AT INFINITY OF POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract:

The picture of the trajectory of the system of differential equations

$$\frac{dx}{dt} = P(x) = \sum_{k=0}^{m} P_n^k(x), \qquad x = (x_1, x_2, \dots, x_n) \ (1)$$

in the field induced by the field P(x) in the ball B^n ,which is a compact manifold, is considered. The issues of classification and coexistence of isolated singular points are studied, as well as issues of classification and coexistence of exceptional directions, singular lines and others.

Keywords: Qualitative picture, trajectory property of a compact sphere, induced field, invariant set, characteristic number, periodic trajectory of singular points, closed ball, equator, central projection.

INTRODUCTION.

In this paper, we consider the picture of trajectories $n \geq 3$ dimensional homogeneous system of differential equations at infinity in the field X (x). To simplify the research at infinity, the field induced by the field X(x) in the ball B^n will be called the generalized Poincare field and denoted by P(x). The field P(x) will turn out to be analytical and its narrowed at the equator S^{n-1} will be polynomial.

Finding all characteristic numbers of the singular points of the equator R(g ^{i}) and periodic trajectories $\nabla(\theta^j)$, system (1). Let us study the stability or instability in the sense of Lyapunov of the singular points g^i and the periodic trajectories θ^j.

In this note, we study the behavior of the trajectories of a system of differential equations of infinity in the field P(x).

$$\frac{dx}{dt} = P(x)$$

$$x = (x_1, x_2, \dots, x_n), \quad P(x) = (P_1(x), P_2(x), \dots, P_n(x))$$

here $P(x)$ – is a homogeneous vector - functions of the k^{th} dimension, class C^v, $v \geq 1$, $P(x)$- is a field defined in R^n, a non-compact manifold.

In the research of the qualitative picture of trajectories, an important role is played by the research of the trajectories behavior at infinity. Here we continue the method of studying the trajectories property of system (1) at infinity.

Let's consider systems (1) when P(x) are polynomials, that is

$$\frac{dx}{dt} = \sum_{p=0}^{m} P_p(x) \qquad (1)$$

$$P_p(x) = (p_{1p}(x), p_{2p}(x), \dots, p_{np}(x))$$

here $P_{kp}(x)$ – is a homogeneous polynomials of the p^{th} dimension of the field are not compact in the plane R^2 [1]. Poincare began studying polynomial vector fields P = (p_1, p_2) on the plane R^2 by the central projection of trajectories at the origin. Poincare's idea is that the study of a field on a compact sphere S^2 follows from the study of a field in a non-compact plane R^2 [1]. This idea is generalized to the case of the n-dimensional space R^n, which will be presented in this article.

Poincare considered S^2 as a differentiable manifold and obtained a field on S^2, which is induced by a field in the plane R^2. The field obtained by the central projection in the upper and gentler hemisphere suffers a discontinuity on the circle S^1, called the equator. However, if the induced field is multiplied by a definite function depending only on the degree m of the components P_i (x) of the field P = (p_1, p_2), from the analytic continuation of the field to the equator is possible. Such an extended field will be called the Poincare field and denoted by P(p). The equator S^2 is for P(p) an invariant set composed of entire trajectories. The restriction (restriction) of the field P(p) to S^1, denoted by $P^{(\infty)}$, is a polynomial field multiplied by an analytic function that does not vanish.

METHODS AND MATERIALS.

A. Poincares above considerations for the plane R^2 can be easily extended to the n-dimensional Euclidean space R^n. Now the set of points at infinity is represented by an n-1 dimensional sphere S^{n-1}, which we call the equator and which is also an invariant set for the field P = (p_1, p_2, …, p_n) or, which is the same thing, for system (1) in R^n the Poincare field on the sphere S^2 and its restriction to $S^{n-1}$are denoted by P(p) and $P^{(\infty)}$, respectively and as before, the field $P^{(\infty)}$on S^{n-1} is a polynomial multiplied by a nonzero analytical factor.

The field P(p) turns out to be analytic and its contraction at the equator S^{n-1} will be polynomial with respect to $(1+r^2)^{\frac{1}{2}}$, r is the Euclidean norm of the vector x.

Consider the Euclidean space R^n and some of its mapping into R_1^n $R^n \to \{ y \in R_1^n |_{\|y\| \le 1}\}$. Consider a ball B^n = $\{ y \in R_1^n |_{\|y\| \le 1}\}$, having a center at the origin 0 with a radius equal to one. Let us find some field P(x) in the ball B^n corresponding to the projected trajectories. Points at infinity R^n. The points at infinity R^n (two for each direction) are now found in one-to-one correspondence with the points of the sphere.

$S^{n-1} = \{ y \in R_1^n |_{\|y\| \le 1}\}$

The sphere S^{n-1} is called the equator for B^n. The sphere Sn - 1 is called the equator for Bn. An essential feature of the vector field P(x) and S^n is that the equator S^{n-1} is always an invariant set, except for the case of a special type at infinity. On the differential manifold B^n consider the coordinate neighborhoods given by the relations:

$$u_i = \{y \in B^n |_{y_i > 0,} i = \overline{1,n} \}, \quad v_i = \{y \in B^n |_{y_i < 0,} i = \overline{1,n} \}$$

We define coordinate maps

$$\Phi : u \to R^n \quad и \quad \psi : v \to R^n$$

by means of the following relations:

$$\Phi(y) = \frac{y}{\sqrt{1 - \|y\|^2}}$$

Then the central projection of the space R^n and the ball B^n puts in correspondence for each point x∈ R^n two points, which are determined by the relations:

$$y = f(x) = \frac{x}{\sqrt{1 - \|x\|^2}}, \quad y = f'(x) = -\frac{x}{\sqrt{1 - \|x\|^2}}$$

here $\|y\|, \|x\|$ – Euclidean norm of vectors y, x .

The field Π_∞(x) induced on the sphere S^{n-1} is defined as follows. We associate the vector P(x) with x∈ $R^n$for the vector df_x(P(x))with y= f (x) and $df_x'((P(x))$ at $y = f(x)$, here df_x and df_x' are total differentials at the point x of the function 〖df_x and df_x' are discontinuous at the equator S^{n-1}. However, if these fields are multiplied by the factor $\left(\sqrt{1 - \|y\|^2}\right)^{m-1}$then the equator S^{n-1} becomes an invariant set.

Theorem 1. The vector field induced by the differentials df and df′ to the complement $B^n|S^{n-1}$ to the equator S^{n-1} can be analytically continued to the ball B^n. The resulting field will have an equator S^{n-1} invariant set, except for a special type at infinity, the ball $B^{-n}$is a closed ball.

We transform system (1) to coordinates y.

$$\begin{cases} \dfrac{dy}{d\tau} = \displaystyle\sum_{k=0}^{m} r_1^{m-k} \Phi^k(y) \\ \dfrac{dr_1}{d\tau} = -r_1 \displaystyle\sum_{k=0}^{m} r_1^{m-k} R_{k+1}(y) \end{cases}, \qquad (2)$$

here $\Phi^K(y) = P^K(y) - yR_{k+1}(y)$, $R_{k+1}(y) = \sum_{i=1}^{m} y_i P_i^k(y)$

$$d\tau = r_1^{-m+1} dt, \quad r_1 = \sqrt{1 - \|y\|^2}$$

The singular points can equator S^{n-1} are found from the solution of the system offset [2].

$$r_1 = 0, \quad \Phi^{m+1}(y) = P^m(y) - yR_{m+1}(y) = 0$$

If the identity

$$\Phi_i^m(y) = 0 \qquad (3)$$

For each j, we assume that we have a special type at infinity.

If system (3) is not satisfied identically for all indices j, then we assume that there is a semi-singular type at infinity.

The singular points can be at the intersection points of the surfaces P(x)= 0, but are determined by the trajectory behavior of system (1) with their unlimited distance.

Let us introduce the following notation and definitions:

1. $R_{m+1}(g)$ – characteristic number of the singular point g ∈ S^{n-1}.
2. $\nabla(\theta) = \int_0^T R_{m+1}(\theta) ds$ – characteristic number of the periodic trajectory $\theta \in S^{n-1}$, где $\theta(S+T) = \theta(S)$.

3. The set of singular points of the vector fields R (x) and P (x), respectively, will be denoted by M and E, M⊂ E.
4. The set of elements E for which $R_{m+1}(g) \neq 0$ does not belong to the set M and we denote by F_1, E = G_EM.
5. The set of elements E for which $R_{m+1}(g) = 0$ s denoted by F_2E = M∪F_1∪ F_2

If S^{n-1} is an invariant set in B^n then the F_2elements will be singular points of higher order, since they appear when the elements of the set M and F_1merge.
If the equator is not an invariant set in B^n, which is possible only for a special type, then F_1= ∅, in this case M = E. Elements of the set F_i are called infinitely distant singular points of system (1).
Theorem 2. The maximum number of singular points of the polynomial system (1) in the ball B^n is equal to (1)

$$2\sum_{i=0}^{n-1} m^i + m^n$$

for the maximum number of singular points of the polynomial system (1) in the ball B^n, the number of elements in the set F_1 is

$$2\sum_{i=0}^{n-1} m^i$$

We know that for the maximum number of singular points of the polynomial system (1) in the ball B^n, the number of elements of the set F_1 is equal to

$$2\sum_{i=0}^{n-1} m^i$$

and the maximum number of solutions to an algebraic equations system is
P(x) + x f^m (x) = 0 (4)
in the real numbers is $\sum_{i=0}^{n} m^i$. The maximum number of solutions to system (4) in the complex numbers is $(m+1)^n - \sum_{i=0}^{n} m^i$,
here P(x) – is a vector polynomial of degree m , $f^m(x)$ – is a form of degree m with constant coefficients.
Let P(x) be a homogeneous vector function of degree m in system (1), that is,
P(**λ**x) = λ^mP(x)
Then system (2) takes the form

$$\begin{cases} \dfrac{dy}{d\tau} = \Phi^m(y) \\ \dfrac{dr_1}{d\tau} = -r_1 R_{m+1}(y) \end{cases} \qquad (5)$$

by γ we denote the trajectories of system (6) on the sphere S^{n-1}. Consider the cone C(γ) formed by the rays passing through the point 0 and the points of the trajectories γ. This surface is an integral manifold of the homogeneous system (1)

RESULTS.

The integral manifolds generated by the system (5) trajectories are disjoint smooth cones C(γ), on each of which it is possible to study the trajectories pattern of system (1).
If by ω - we denote the trajectories of system (1), then $\omega = r_1(\gamma)\cdot\gamma$, where r $_1(\gamma)$is the solution to equation (6) determined by the formula:

$$r_1(\gamma) = -r_1^0 exp\int_{-\tau}^{\tau} R(\gamma)ds$$

If the trajectory γ is an isolated singular point of system (5), then the integral cone degenerates the integral rayλg, **λ** > 0, and lemma 1 holds.
Lemma 1. In order for the integral system (1) to have T - periodic trajectories on the cone G(θ), it is necessary and sufficient that system (5) has a T - periodic trajectory θ with a characteristic number equal to zero.
Let system (5) be rough in the sense of Morse-Smale, then the following theorem holds.
Theorem 3. For the singular point 0 of the homogeneous system (1) to be stable in the Lyapunov sense, it is necessary and sufficient that for all characteristic numbers of the singular points of the S^{n-1}, $R_{n+1}(g^i) > 0$ equator and for periodic trajectories θ^i , $\nabla(\theta^j) \geq 0$.
Theorem 4. For the singular point 0 of the homogeneous system (1) to be unstable in the Lyapunov system, it is necessary and sufficient that the characteristic numbers $R_{n+1}(g^i)$ and $\nabla(\theta^j)$, the singular points of the periodic trajectories of the S^{n-1} equator have at least one pair of opposite signs.

CONCLUSION.

In this study, a qualitative picture shows the study of the behavior of trajectories in infinity (1) system. Some field P(x) in the ball B^n corresponding to the projected trajectories. Points at infinity R^n

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