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AN ITERATIVE ALGORITHM FOR CONSTRUCTING A DELAUNAY TRIANGULATION

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Article history:		Abstract:
Received:	3rd May 2022	For the first time the task of constructing the Delaunay triangulation was posed in 1934 in the work of the Soviet mathematician B.N. Delaunay.
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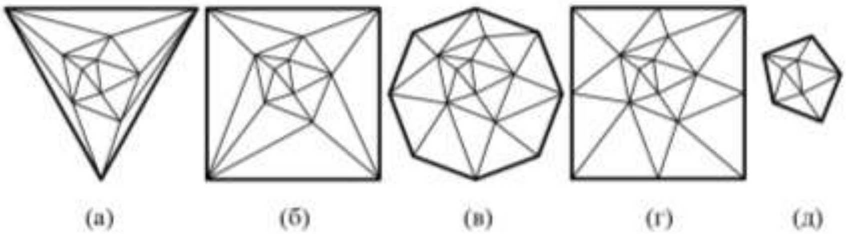
There are algorithms that achieve this estimate in the average and worst cases. In addition, algorithms are known that allow in some cases to achieve on average (θ√N). For further discussion, we introduce several definitions: 1. A planar graph is called a triangulation, all of whose internal regions are triangles. 2. A convex triangulation is such a triangulation for which the minimum polygon enclosing all triangles will be convex. 3. A triangulation that is not convex is called non-convex. The problem of constructing a triangulation from a given set of two-dimensional points is the problem of connecting given points with non-intersecting segments so that a triangulation is formed. The task of constructing a triangulation based on the initial set of points is ambiguous, so the question arises, which of the two different triangulations is better? 4. A triangulation is called optimal if the sum of the lengths of all edges is minimal among all possible triangulations built on the same starting points. It is substantiated that the problem of constructing such a triangulation is apparently NP-complete. Therefore, for most real problems, the existing algorithms for constructing optimal triangulation are unacceptable due to too high labor intensity [1]. If necessary, approximate algorithms are used in practice. One of the first proposed the following algorithm for constructing triangulation. Greedy triangulation algorithm. Step 1. A list of all possible segments connecting pairs of initial points is generated, and it is sorted by segment lengths. Step 2. Starting from the shortest one, segments are sequentially inserted into the triangulation. If the segment does not intersect with other previously inserted segments, then it is inserted, otherwise it is discarded[2]. End of the algorithm. Note that if all possible segments have different lengths, then the result of this algorithm is unambiguous, otherwise it depends on the insertion order of segments of the same length [3]. 5. A triangulation is called greedy if it is built by a greedy algorithm. In addition to optimal and greedy triangulation, Delaunay triangulation is also widely known, which has a number of practically important properties. 6. It is said that triangulation satisfies the Delaunay condition if none of the given triangulation points falls inside the circle circumscribed around any constructed triangle. 7. A triangulation is called a Delaunay triangulation if it is convex and satisfies the Delaunay condition. All iterative algorithms are based on a very simple idea of successively adding points to a partially constructed Delaunay triangulation. Formally, it looks like this. An iterative algorithm for constructing the Delaunay triangulation. Given a set of N points. Step 1. On the first three starting points we build one triangle[4]. Step 2. In the loop over n, for all other points, perform steps 3–5. Step 3. The next n-th point is added to the already constructed triangulation structure as follows. First, the point is localized, i.e. there is a triangle (constructed earlier), in which the next point falls. Or, if the point does not fall inside the triangulation, there is a triangle on the border of the triangulation, closest to the next point[5]. Step 4. If a point hits a previously inserted triangulation node, then such a point is usually discarded, otherwise the point is inserted into the triangulation as a new node. Moreover, if the point hits some edge, then it is divided into two new ones, and both triangles adjacent to the edge are also divided into two smaller ones. If the point is strictly inside any triangle, it is divided into three new ones. If the point is outside the triangulation, then one or more triangles are built[6]. Step 5. Local checks of the newly obtained triangles for compliance with the Delaunay condition are carried out and the necessary rearrangements are performed. End of the algorithm. The complexity of this algorithm consists of the laboriousness of finding a triangle, in which a point is added at the next step, the laboriousness of constructing new triangles, as well as the laboriousness of the corresponding rebuilding of the triangulation structure as a result of unsatisfactory checks of pairs of neighboring triangles of the obtained triangulation for the fulfillment of the Delaunay condition [7]. When constructing new triangles, two situations are possible when the added point falls either inside the triangulation or outside it. In the first case, new triangles are constructed and the number of actions performed by the algorithm is fixed. In the second, it is necessary to build additional triangles external to the current triangulation, and their number can be 3n in the worst case – However, no more than 3 N · triangles will be added for all steps of the algorithm, where N is the total number of initial points. Therefore, in both cases, the total time spent on the construction of triangles is (θ√N). In order to somewhat simplify the algorithm, we can generally get rid of the second case by first introducing into the triangulation several such additional nodes that the triangulation built on them will certainly cover all the initial points of the triangulation. Such a structure is usually called a superstructure [8]. In practice, the following options are usually chosen for the super structure (Fig.



a - a triangle; b - square; c - points on the circle; d - points on the square; e-convex hull. a) the vertices of an equilateral triangle covering the entire set of initial points (Fig. a); b) the vertices of the square covering the entire set of initial points (Fig. b); c) (θ $\sqrt{N}$)points uniformly distributed along the circle covering the entire set of initial points (Fig . c); d) (θ $\sqrt{N}$)points uniformly distributed along the square covering the entire set of initial points (Fig. d); e) initial points falling on the convex hull of the set of initial points (Fig. e). The results of an experimental comparison of various versions of superstructures are presented [9]. At the same time, it is shown that when using a superstructure on different distributions of the initial data, it is possible both to increase the speed of the algorithms and to reduce it, but not more than by 10%. Any addition of a new point to the triangulation can theoretically violate the Delaunay conditions, so after adding a point, the triangulation is usually immediately checked locally for the Delaunay condition. This check should cover all newly built triangles and their neighbors. The number of such rebuildings in the worst case can be very large, which, in fact, can lead to a complete rebuilding of the entire triangulation. Therefore, the complexity of rebuildings is ($O\sqrt{N}$. However, the average number of such rebuildings on real data is only about three. Thus, the procedure for finding the next triangle makes the greatest contribution to the complexity of the iterative algorithm. That is why all iterative algorithms for constructing the Delaunay triangulation differ almost only in the procedure for finding the next triangle[ten].

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