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ROTATION SURFACE SECTIONS OF THE SECOND ORDER ACCORDING TO A GIVEN ELLIPSE.

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Abstract:
The article considers the question of finding the position of planes crossing surfaces of revolution of the second order along an ellipse of a shape given by parameters. Ellipsoids, paraboloids, one-sheet hyperboloids, and cones are considered as examples of surfaces of revolution. The shape of the ellipse is the major and minor axis of the curves.

Keywords: Ellipse, rotations, ellipsoid, paraboloid, one-sheeted hyperboloid, cone, large, small, axes, envelope, frontal projection, tangent, surface, symmetrical, conjugate.

INTRODUCTION.

To determine the position of the desired plane, we draw two planes **Q** and **Q'**, located from the axis of the surface at a distance equal to the semi-axes of the given ellipse b_1 or a_1, parallel to the **XOZ** plane. Then on the **XOZ** plane, projections of two coincident curves β_2 and β'_2 are obtained, which are the locus of projections of the ends of the minor axes, given ellipses. Then it is enough to draw such a front-projecting plane so that, firstly, it intersects the frontal outline of the surface along a segment equal to $2a_1$ or $2b_1$, and, secondly, touches the middle of this segment with the curve β_2. Curve β_2 is symmetrical with respect to the axis o_2z_2. Therefore, a second plane can also be drawn, which intersects the o_2z_2 axis at the same point as the first plane. If we take this point as the vertex of the cone, and any of the traces of the planes as the generatrix, we obtain a frontal projection of the cone of revolution coaxial with the given surface.
Any plane tangent to this cone of revolution intersects the given surface in a given ellipse. This direction is easy to determine analytically.

1. Ellipsoid of revolution $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{c^2}=1.$ **(1.1)**
cross the plane along an ellipse whose semi axes are equal to a_1 and b_1.
Intersecting the ellipsoid by planes **Q** and **Q'** parallel to the **XOZ** plane, located at a distance b_1 from the OZ axis, we obtain an ellipse:

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=\frac{a^2-b_1^2}{a^2} \qquad \textbf{(1.2)}$$

This ellipse, or curve β_2, is similar to the frontal outline of the ellipsoid **(1.1).**

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1. \qquad \textbf{(1.3)}$$

Let the equation of the frontal projection of one of the generators of the cone or the frontal trace of the plane be $z=kx+t.$ **(1.4)**
where **k** is the angle of inclination of the generatrix to the **XOY** plane.
Solving (1.3) and (1.4) together, we obtain

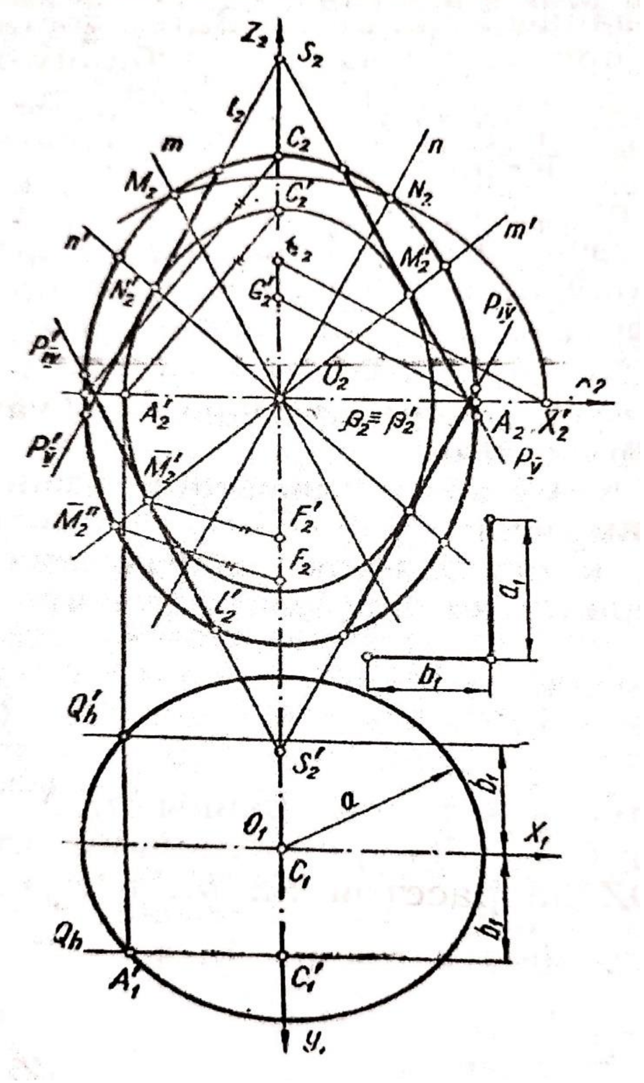
$$x_{1,2}=\frac{-a^2kt\pm ac\sqrt{c^2+a^2k^2-t^2}}{c^2-a^2k^2}, \qquad z_{1,2}=\frac{c^2t\pm ack\sqrt{c^2+a^2k^2-t^2}}{c^2+a^2k^2} \qquad \textbf{(1.5)}$$

From the above conditions, it turns out $(x_1 - x_1)^2+(z_1 - z_1)^2=(2a_1)^2.$ **(1.6)**

и $\frac{x_0^2}{a^2}+\frac{z_0^2}{c^2}=\frac{a^2-b_1^2}{a^2}$ in which $x_0=\frac{x_1+x_2}{2}$ and $z_0=\frac{z_1+z_2}{2}$ **(1.7)**

Solving (**1.6**) and (**1.7**) together, we obtain the slope of the cone generatrix

$$k=\pm\sqrt{\frac{b_1^2-a_1^2}{\left(\frac{aa_1}{c}\right)-b_1^2}}. \qquad \textbf{(1.8)}$$



Using the geometric interpretation of the slope **k**, you can determine the direction of the generatrix of the cone. But using the similarities between the required section and a section parallel to it, drawn through the center of the surface at the frontal outline of the ellipsoid, one can find a diameter whose slope will also be equal to **k**. To build such a diameter on the o_2z_2 axis, debug the segments $a_1=O_2G_2$ and $b_1=O_2G'_2$ (Fig. 1).

Connecting the point G'_2 with the point A_2, we draw a straight line through G_2 parallel to G'_2A_2 until it intersects with o_2x_2 at the point x_2. With a radius of o_2x_2, we draw a circle that intersects the frontal outline of the ellipsoid at points N_2 and M_2. The slope coefficients of the obtained diameters n and m are equal to the slope coefficients of the generatrices of the desired cone. We build the diameters n' and m', respectively, conjugate n and m. we determine the points of contact N'_2 and M'_2 of the generatrix of the cone with the curve β_2. Through these points we draw a front-projecting plane parallel to n and m, intersecting the ellipsoid along a given ellipse. At the same time, we obtain the desired cone with vertex S_2 and S'_2 and generators l_2 and l'_2.

Using the similarity of ellipses **(1.2)** and **(1.3)**, one can prove that spices $F'_2M'_2$ and F_2M_2 are parallel to each other. F'_2 and F_2 are the foci of the ellipses **(1.2)** and **(1.3)**.

This makes it possible, without drawing the curve β_2, to determine the position of the desired plane.

2. Paraboloid of revolution $\frac{x^2}{p}+\frac{y^2}{p}=2z$. **(2.1.)**

cross along an ellipse whose semi axes are a_1 and b_1.

Intersecting the paraboloid **(2.1.)** by the plane **Q||XOZ**, located at a distance b_1 from the **OZ** axis, we obtain the parabola β_2, which is the locus of projections of the ends of the minor axes $x^2=2pz-b_1$. **(2.2.)**

This parabola is congruent with the frontal outline of the paraboloid **(2.1.)**.

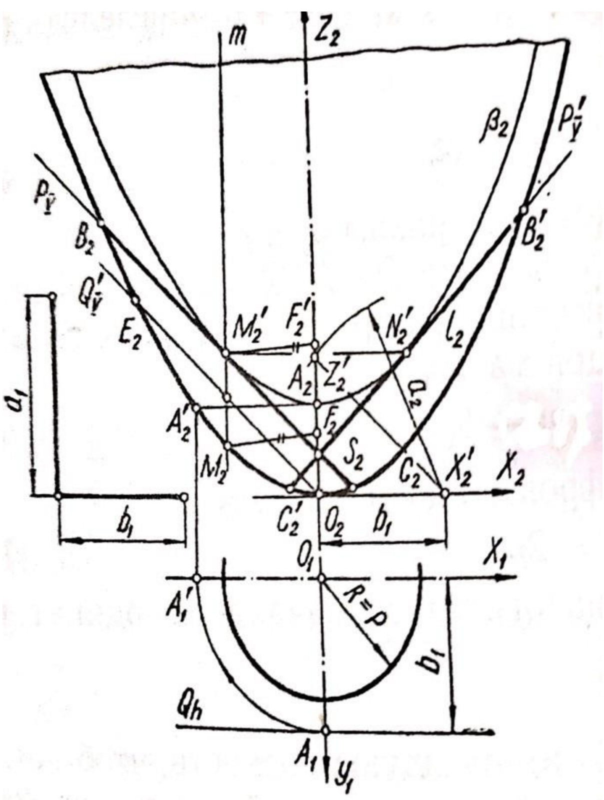
$x^2 = 2pz$. **(2.3.)**

Its vertex is located at a distance from the origin of coordinates $Z=\frac{{b_1}^2}{2p}$.

We build such a frontally projecting plane so that, firstly, it intersects the frontal outline of the paraboloid along a segment equal to $2a_1$, and, secondly, touches the middle of this segment with curve **(2.2.)**.

Similarly, to the first example, analytically we find the slope

$$k=\pm\frac{1}{b_1}\sqrt{a_1^2-b_1^2}. \qquad (2.4.)$$



To obtain the direction of the generatrix of the cone from the point **o_2** on the axis **o_2x_2**, postponing the semi-axis **b_1** (Fig. 2), we obtain the point **x'_2.** With a radius **a_1** of their center **x'_2**, we make a notch on the axis **o_2z_2.** Connecting the points **z'_2** and **$x'2$**, we obtain the direction of the generatrix of the desired cone. To obtain a tangent point on the curve **β_2**, we draw the diameter m conjugate to the found slope. The point **M'_2** of the intersection of m and the curve **β_2** is the point of contact. Through **M'_2** and **N'_2**, which is symmetric to it, one can draw traces of the project planes **P** and **P'** intersecting the paraboloid **(2.1.)** along a given ellipse. At the same time, we obtain a cone with vertex **S_2** and generatrix **l_2**.

It is easy to prove that the lines connecting the foci of the parabolas **(2.2.)** and **(2.3.)** with the intersection points of diameter m with these parabolas are parallel to each other. This makes it possible to solve the problem without drawing the curve **β_2.**

3. One-sheeted hyperboloid of revolution $\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2}$ **= 1. (3.1)**

intersect in an ellipse whose semi axes are known.

Crossing the one-sheeted hyperboloid by the plane **$y= b_1$**, we obtain the frontal projection **β_2** of the curve **β**, the equation of which will be

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = \frac{a^2 - b_1^2}{a^2} \qquad \textbf{(3.2)}$$

This hyperbola is similar to the frontal outline $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$ **(3.3)**

of the hyperboloid of one sheet **(3.1)** and has common asymptotes. To determine the direction of the generatrices of a cone with a radius **$o_2x'_2$**, we draw a circle with a center **o_2** (Fig. 3). Then we build the directions **n** and **m**, which have the same slope with the generatrix of the cone

$$k = \pm\sqrt{\frac{a_1^2 - b_1^2}{b_1^2 + \left(\frac{aa_1}{c}\right)}}2. \qquad \textbf{(3.4)}$$

To determine the point of contact **N_2, N'_2** and **M_2, M'_2,** we construct the diameters **n'** and **m'**, which are conjugate to **n** and **m**, respectively.

4. To determine the position of planes intersecting the cone of revolution

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = 0. \qquad \textbf{(4.1)}$$

for a given ellipse, as in the previous examples, we analytically find the direction of the generatrix of the desired cone. The slope of this direction is equal to

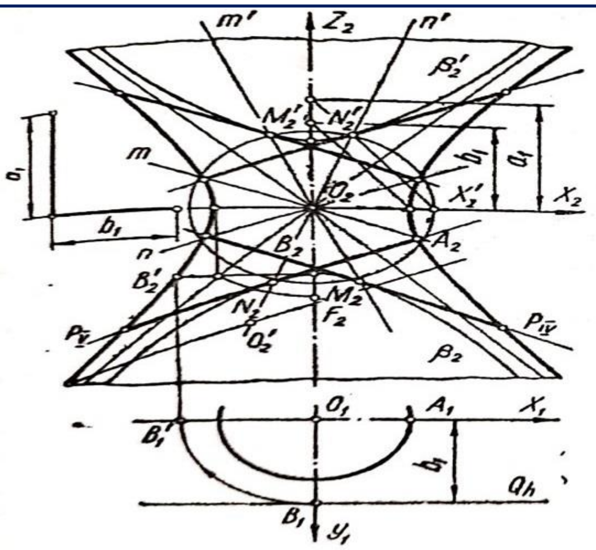
$$k = \pm\sqrt{\frac{a_1^2 - b_1^2}{b_1^2 + \left(\frac{aa_1}{c}\right)}}\ 2. \qquad \textbf{(4.2)}$$

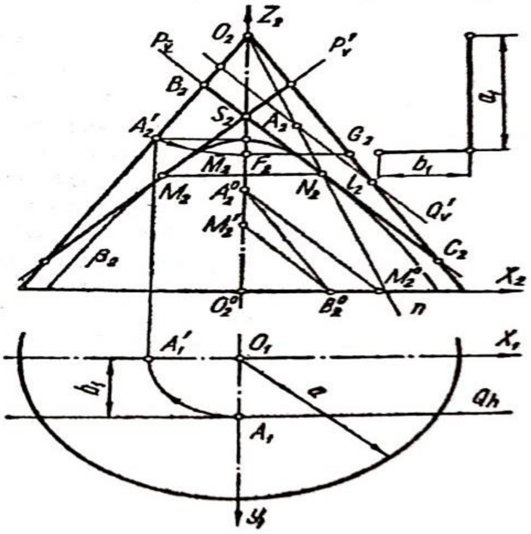
To construct a direction, we will proceed as follows: construct a segment $o_2^0 A_2^0 = \sqrt{a_1^2 - b_1^2}$ (Fig 4), line segment ***M_2G_2*=**$\frac{aa_1}{c}$ and line segment $B_2^0 M_2' = \sqrt{b_1^2 + \left(\frac{aa_1}{c}\right)^2}$. **(4.3)**

From the point o_2^0 we plot the segment $B_2^0 M_2'$on the axis ***o_2x_2*** and obtain the point M_2^0. Connecting the points A_2^0 and M_2^0, we obtain the direction of the trace of the deep tangent plane.

To find the point of contact of the plane of the desired ellipse with the curve β_2, we construct a straight line ***n***, which is the frontal projection of the projecting plane, to which the centers of the ellipses of the found slope belong. The intersection point ***N_2*** of the line ***n*** with the curve β_2 is the point of contact.

Similarly, as in the previous examples, one can determine the positions of planes intersecting a two-sheeted hyperboloid along a given ellipse.





Thus, the above methods can solve problems related to finding the position of a plane intersecting surfaces of revolution of the 2nd order along a predetermined ellipse, which is often encountered in design practice.

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