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DOMINATING SETS AND DOMINATION POLYNOMIAL OF CORONA OF GRAPHS

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Abstract:

Let $G = (V, E)$ be a simple and undirected grap. A subset D of V is called dominating set of G, if $\forall\, v \in V$ either v in D or is adjacent to at least one vertex in D. Let G_m be corona graph $G_m = G_n \odot K_1$ with order $m = 2n$. In this paper, the G_m^i, is constructed and the recursive formula for $d(G_m, i)$ is obtained. The polynomial $D(G_m, x) = \sum_{i=1}^{m} d(G_m, i)x^i$, (domination polynomial) for corona graph with some properties of this polynomial is determined

Keywords: Dominating set, Family of dominating sets, Domination Polynomial, Corona of Graph

1. INTRODUCTION

Assume that $G = (V, E)$ be a simple and undirected graph. The subset D of V is called a dominating set of G if every vertex in set $V - D$ is adjacent to at least vertex in D.The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G [12].The subject of domination in graph theory of the statement appealed to many researchers, including them[1-9] by putting some condition on set V. As well as from research in the study of domination polynomials [10-16], and others (Chromatic Polynomials) [21,22]. In [10] C. Berge is the first introduced the domination parameter. The equality co-neighborhood, inverse equality co-neighborhood, total equality co-neighborhood domination, fuzzy equality co-neighborhood domination and strong equality co-neighborhood domination are introduced in [17,18,19,23]. Let G_n^i be the family of dominating sets of a graph $G_n$with cardinality i and let $d(G_n, i) = |G_n^i|$. The polynomial $D(G_n, x) = \sum_{i=r(G)}^{n} d(G_n, i)x^i$, is defined to be the domination polynomial of a graph G [12]. In this paper, the families of dominating sets of corona graph $G_m = G_n \odot K_1$ with cardinality i. $d(G_m, i)$ are constructed. In addition, the domination polynomial of G_m $(D(G_m, x) = \sum_{i=n}^{m} d(G_m, i)x^i)$ is investigated. As usual we use $\binom{n}{i}$ for the combination n to i.

Definition 1.1. [24]
The corona $(G_1 \odot G_2)$ of two graphs G_1 and G_2 is the graph obtained by taking one copy of G_1 (which has $|V(G_1)|$) copies of G_2, where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. Let $G_1 \equiv G_n$ and $G_2 \equiv K_1$, $(G_n \odot K_1)$. *(see Fig .1)*

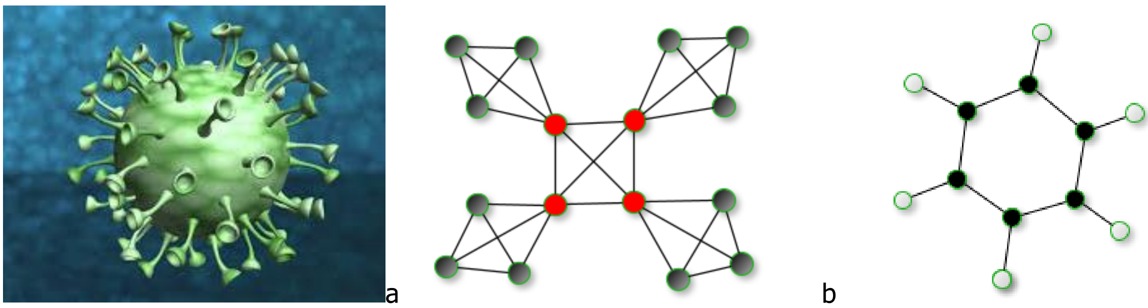

Figure 1: (a) $K_4 \odot C_3$ **(b)** $C_6 \odot K_1$

To prove our main results we need the following lemmas:

Lemma 1.2. [11]. The following properties hold for all graph G.such that $d(G_n, i) = |G_n^i|$.
(i) $|G_n^i| = 0$ if $i > n$ (ii) $|G_n^n| = 1$ (iii) $|G_n^0| = 1$ $\forall\, n \geq 0$ (iv) $|G_n^{n-1}| = n$

Lemma 1.3. [11]. The following properties hold for all $n \geq 0$.
(i) $\binom{n}{i} = 0$ if $i > n$ (ii) $\binom{n}{n} = 1$ (iii) $\binom{n}{0} = 1$ $\forall\, n \geq 0$

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Theorem 1.4. [11]. Let S_n be star with order n, then

$d(S_n,i) = \binom{n}{i} - \binom{n-1}{i}$ ∀ i < n -1, n ≥3

Theorem 1.5. [11]. Let S_n be star with order n, then

$d(S_n,i) = \binom{n-1}{i-1}$ ∀ i < n -1, n ≥3

2 dominating sets of corona graphs

In this section, $d(G_m,i)$ of $(G_n \odot K_1)$, is investigated

Theorem 2.1. Let $C_m = (G_n \odot K_1)$ be corona graph, then

$$d(G_m,i) = \sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n} \quad \forall\, n \geq 1\,.$$

Proof.

Let G_m be corona graph $G_m = G_n \odot K_1$ with order $m = 2n$, and let D is dominating set, then we have n of the copies of K_1 accordant to Definition 1.1. in this case we have n of the pendant vertices therefore must be every vertex in G_n or the pendant vertex to which it is adjacent must be belong to D, then $|D| = i \geq n$. Now if we take n-k of the vertices from the first graph G_n , in this case we must reserve k vertices from the pendant vertices, so we have n-k from the lone vertices, so the probabilities of the first graph $\binom{n}{n-k}$ and the probabilities of the pendant vertices are $\binom{n-k}{i-n}$, and as a result, the number of dominating set for each k are $\binom{n}{n-k}\binom{n-k}{i-n}$, and since k = 0,1,...n, then the total number of each dominating sets is $\sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n}$ for all cardinality $i \geq n$ ■

Theorem 2.2. Let $C_m = (G_n \odot K_1)$ be corona graph, then

$$d(G_m,i) = 2^n \quad \forall\, i = n\,.$$

Proof.

Let G_m be corona graph $G_m = G_n \odot K_1$ with order $m = 2n$, then $d(G_m,i) = \sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n}$ $\forall\, n \geq 1$ according to Theorem 2.1. If $n = i$, then $d(G_m,i) = \sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{0} = \sum_{k=0}^{n} \binom{n}{n-k}$, because $\binom{n-k}{0} = 1$ according to Lemma 1.3. Now to prove $\sum_{k=0}^{n} \binom{n}{n-k} = 2^n$ by using mathematical induction.

1. Let $n = 1$ to prove $\sum_{k=0}^{1} \binom{1}{1-k} = 2^1$, we have $\sum_{k=0}^{1} \binom{1}{1-k} = \binom{1}{1-0} + \binom{1}{1-1} = 1 + 1 = 2$, then the relationship is true when $n = 1$.
2. Suppose that the relationship is true when $n = r$, then $\sum_{k=0}^{r} \binom{r}{r-k} = 2^r$
3. To prove that the relationship is true when $n = r + 1$. We have $\sum_{k=0}^{n} \binom{n}{n-k} = \sum_{k=0}^{r+1} \binom{r+1}{r+1-k} = \sum_{k=0}^{r+1} \binom{r}{r+1-k} + \binom{r}{r-k} = \sum_{k=0}^{r+1} \binom{r}{r+1-k} + \sum_{k=0}^{r+1} \binom{r}{r-k} = \sum_{k=1}^{r+1} \binom{r}{r+1-k} + \sum_{k=0}^{r} \binom{r}{r-k} = \sum_{k=0}^{r} \binom{r}{r-k} + \sum_{k=0}^{r} \binom{r}{r-k} = 2^r + 2^r = 2(2^r)$ $= 2^{r+1}$ according to Theorem 1.4 and Theorem 1.5 and Lemma 1.3. therefore the relationship is true when $n = r + 1$. Thus the proof is done. ■

Let $G_m = (G_n \odot K_1)$ be a corona graph with order 2*n*. Using Theorem 2.1. and Theorem 2.2. obtain the coefficients of $D(G_m,x)$ for 1≤ *n* ≤ 10 in Table 1. Let $d(G_m,i) = |G_m^i|$. There are interesting relationships between the numbers $d(G_m,i)$ (1≤ *i*≤ *2n*) in the table.

Table 1. $d(G_m,i)$The number of dominating set of *corona graph* $C_m = (G_n \odot K_1)$ with cardinality i

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n																				
1	2	1																		
2	0	4	4	1																
3	0	0	8	12	6	1														
4	0	0	0	16	32	24	8	1												
5	0	0	0	0	32	80	80	40	10	1										
6	0	0	0	0	0	64	19	240	160	60	12	1								

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							2													
7	0	0	0	0	0	0	12 8	440	672	560	280	84	14	1						
8	0	0	0	0	0	0	0	256	1024	1792	1792	1120	532	112	16	1				
9	0	0	0	0	0	0	0	0	512	2304	4626	5376	4032	2016	672	144	18	1		
10	0	0	0	0	0	0	0	0	0	1024	5120	11520	15360	13440	4032	3360	960	180	20	1

In the following theorem, we obtain some properties of $d(G_m, i)$ of $C_m = (G_n \odot K_1)$

Proposition 2.3. Let $C_m = (G_n \odot K_1)$ be corona graph, then

(i) $d(G_m, i) = 0 \quad \forall\, i < n$
(ii) $d(G_m, i) = 2n$ *if* $i = 2n-1$
(iii) $d(G_m, i) = 1$ *if* $i = 2n.$
(iv) $\gamma(G_m) = n$

Proof.
(i) It is verified through the proof of the Theorem 2.1.
(ii) Since $m = 2n$, then it is verified according to Lemma1.2.
(iii) Since $m = 2n$, then it is verified according to Lemma 1.3.
(iv) Since $d(G_m, i) = 0 \quad \forall\, i < n$ *and* $d(G_m, i) = 2^n \quad \forall\, i = n$ according to (i) and Theorem 2.2, then $\gamma(G_m) = n$. ■

3 domination polynomial of G_n of graphs

In this section, we introduce and investigate the new parameter domination polynomial of G_m.

Definition 3.1.[11] *Let* G_m^i *be the family of* dominating set *of corona graph* $C_m = (G_n \odot K_1)$ *with cardinality i, and let* $d(G_m, i) = |G_m^i|$. *Then the domination polynomial* $D(G_m, x)$ *of* $C_m = (G_n \odot K_1)$ *is defined as* $D(G_m, x) = \sum_{i=1}^{m} d(G_m, i)x^i = \sum_{i=r(G)}^{m} d(G_m, i)x^i$.

In the following corollary, we obtain some properties of $D(G_m, x)$ *of* G_m.

Corollary 3.2.
The following properties of $D(G_m, x)$ *are hold* $\forall\, m \geq 2$

1. $D(G_m, x) = D(G_{2n}, x) = \sum_{i=n}^{2n} d(G_{2n}, i)x^i$
2. $D(G_m, x) = \sum_{i=n}^{2n}\left(\sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n}\right)x^i$
3. $D(G_m, x) = (2x)^n + \sum_{i=n+1}^{2n-2}\left(\sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n}\right)x^i + 2nx^{2n-1} + x^{2n}$

Proof.
From definition of the domination polynomial $D(G_m, x) = \sum_{i=1}^{m} d(G_m, i)x^i = \sum_{i=r(G)}^{m} d(G_m, i)x^i$, we get :

1. Since $m = 2n$ and $\gamma(G_m) = n$, then $D(G_m, x) = D(G_{2n}, x) = \sum_{i=n}^{2n} d(G_{2n}, i)x^i$ according to Definition 1.1. and Proposition 2.3.
2. $D(G_m, x) = \sum_{i=r(G)}^{m} d(G_m, i)x^i = \sum_{i=n}^{2n}\left(\sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n}\right)x^i$, according to Theorem 2.1. and Definition 3.1.
3. Since $d(G_m, i) = 2^n \quad \forall\, i = n$. according to Theorem 2.2 and since $d(G_m, i) = 2n$ *if* $i = 2n-1$ *and* $d(G_m, i) = 1$ *if* $i = 2n$ according to Proposition 2.3. *then* $D(G_m, x) = \sum_{i=n}^{2n} d(G_{2n}, i)x^i = (2x)^n + \sum_{i=n+1}^{2n-2}(d(G_{2n}, i))x^i + 2nx^{2n-1} + x^{2n} = (2x)^n + \sum_{i=n+1}^{2n-2}\left(\sum_{k=0}^{n} \binom{n}{n-k}\binom{n-k}{i-n}\right)x^i + 2nx^{2n-1} + x^{2n}$ according to Definition 3.1. ■

Example 3.3.
Let $G_8 = (K_4 \odot K_1)$ be corona graph, *we can get on* $D(G_8, x)$ *from the table 1. We have* $D(G_8, x) = \sum_{i=4}^{8} d(G_8, i)x^i = 16x^4 + 32x^5 + 24x^6 + 8x^7 + x^8$ *(by Corollary 1). (see Fig* .2(a))

Example 3.4.
Let $G_{18} = (C_9 \odot K_1)$ be corona graph, *we can get on* $D(G_{18}, x)$ *from the table 1. We have* $D(G_{18}, x) = \sum_{i=9}^{18} d(G_{18}, i)x^i = 512x^9 + 2304x^{10} + 4626x^{11} + 5376x^{12} + 4032x^{13} + 2016x^{14} + 672x^{15} + 144x^{16} + 18x^{17} + x^{18}$ *(by Corollary 1). (see Fig* .2(b))

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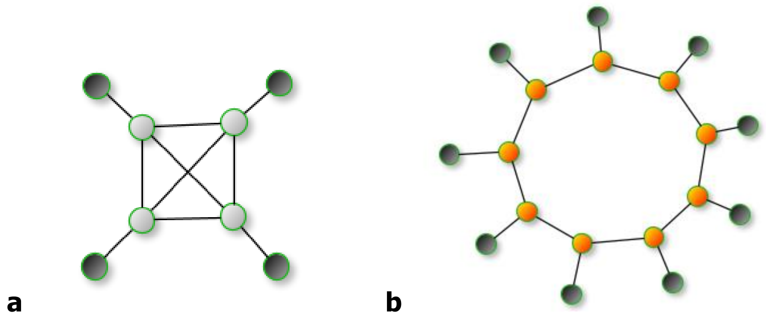

Figure 2: (a) $K_4 \odot K_1$ (b) $C_9 \odot K_1$

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