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ON ONE METHOD FOR SOLVING DEGENERATING PARABOLIC SYSTEMS BY THE DIRECT LINE METHOD WITH AN APPENDIX IN THE THEORY OF FILRATION

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Abstract:

In this article, we will study the problem arising in the study of the processes of diffusion or filtration of a liquid (gas) in multilayer layers, taking into account convective transfer

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In this article, we will study the problem arising in the study of the processes of diffusion or filtration of a liquid (gas) in multilayer layers, taking into account convective transfer [1,2,3].

In the case of a 3-layer reservoir, the problem is formulated as follows:
Define in

$$\overline{D} = \{0 \le x \le 1,\ 0 \le z \le 1,\ 0 \le t \le T\}\ u\ D_1 = \{0 \le x \le 1,\ 0 \le t \le T\}$$

Correspondingly, continuous functions $u(x,t),\ u_1(x,z,t), u_2(x,z,t)$ satisfying the system of equations:

$$\begin{cases} \dfrac{1}{m(x)}\dfrac{\partial}{\partial x}(k(x)\dfrac{\partial u}{\partial x}) = A(x,t)\dfrac{\partial u}{\partial t} + B(x,t)K(x)\dfrac{\partial u}{\partial x} + \sum\limits_{i=1}^{2} A_i(x)K_i(z)\dfrac{\partial u_i}{\partial z}\Big|_{z=1} + f(x,t,u) + \int\limits_0^t R(t,s)u(x,s)ds \\ \dfrac{1}{m_i(z)}\dfrac{\partial}{\partial z}(K_i(z)\dfrac{\partial u_i}{\partial z}) = A_i(z,t)\dfrac{\partial u_i}{\partial z} + f(x,z,t,u_i) \end{cases} \qquad (1)$$

initial conditions $u(x,0) = \varphi(x),\ u_i(x,z,0) = \varphi_i(x,z)$ (2) and boundary conditions

$$\begin{cases} (K(x)\dfrac{\partial u}{\partial x} - a_1(t)u)\Big|_{x=0} = 0 \\ (K(x)\dfrac{\partial u}{\partial x} + a_2(t)u)\Big|_{x=1} = 0 \\ (K_2(z)\dfrac{\partial u_2}{\partial z} - a_{12}(t)u_i)\Big|_{z=0} = 0, \\ u(x,t) = u_i(x,1,t), \qquad i = 1,2 \end{cases} \qquad \text{если} \int\limits_0^1 \frac{dz}{K_i(z)} < +\infty \qquad (3)$$

Here
$F(x,t,u),\ F_i(x,z,t,u_i), \varphi(x), \varphi_i(x,z), K(x), m(x), K_i(x), A(x,t), A_i(z,t), B(x,t), A_2(x), a_k(t),\ (k=1,2), R(t,s)\ a_{12}(t)$
given functions, and
$K(x), m(x) > 0,\ A(x,t) \ge A > 0,\ A_i(z,t) \ge A_{i0} > 0,\ A_i(x) > 0, K_i(0) = 0,\ K_i(z)\ u\ m_i(z)$ - positive for $z \to 0$.
We will assume that the solutions themselves and all known functions in the equations are smooth

If a $\int_0^1 \frac{dz}{K_i(z)} = +\infty, \quad \int_0^1 \frac{\int_0^z m_i(\xi)d\xi}{K_2(z)} < +\infty$, then the condition $(K_i(z)\frac{\partial u_1}{\partial z} - a_{12}(t)u_i)|_{z=0} = 0$ is replaced by the condition $\left|u_i(x,z,t)|_{z=0}\right| < +\infty, \ i = 1,2$.

The peculiarity of these problems is that the desired function enters the equations of the problem in such a way that each of the equations has a "Main" unknown function, while the rest are either not contained or are represented by their own boundary conditions.

Introducing division $t_j = j\tau, \quad j = 0,1; \ N = \left[\frac{T}{\tau}\right]$ we will look for an approximate solution $\{u_j(x)\ u\ u_{ij}(x,z)\}$ using Rote schemes

$$\begin{cases} Lu_j = A(x,t_j)\delta_{\bar{\tau}}u_j + B(x,t_j)K(x)\dfrac{du_t}{dx} + \sum\limits_{i=1}^{2} A_i(x)K_i(z)\dfrac{\partial u_{ij}}{\partial z}\Big|_{z=1} + f(x,t_{j,}u_{j-1}) + \tau\sum\limits_{i=0}^{j-1} R_{j,i}u_i \\ L_z u_{ij} = A_i(z,t_j)\delta_{\bar{\tau}}u_{ij} + f_i(x,z,t_j,u_{ij-1}) \end{cases} \quad (3)$$

$$u_0(x) = \varphi(x), \ u_{i0}(x,z) = \varphi_i(x,z), \ i = 1,2$$

$$\left|\bar{K}(x)\frac{dv_j}{dx} - a_1(t_j)u_j)\right|_{x=0} = 0, \quad (K(x)\frac{du_j}{dx} + a_2(t_j)u_j)|_{x=1} = 0$$

$$(K_i(z)\frac{\partial u_{ij}}{\partial z} - a_{12}(t_j)u_{ij})\Big|_{z=0} = 0, \ u_{ij}(x,z) = u_j(x) \quad (4)$$

where $\delta_{\bar{t}}U_j = \dfrac{U_j - U_{j-1}}{\tau}, \ Lu_j = \dfrac{d}{m(x)}\dfrac{d}{dx}(K(x)\dfrac{dU_j}{dx}), \ l_z u_{ij} = \dfrac{1}{m_i(z)}\dfrac{\partial}{\partial z}(k(z)\dfrac{\partial u_{ij}}{\partial z})$

task for j everyone (3) (4)- linearly relative $\{u_j(x), u_{ij}(x,z)\}$ and has the only solution[1,2,3]. Introducing the norm $\|u\|_j \le \max\limits_{1\le k\le i}|u_k|\ u\ \|\cdot\| = \max|\cdot|$ according to the maximum principle, the evaluation [2]

$$\{|u_j|,|u_{ij}|\} \le \max\left\{\left\|\frac{\frac{A(x,t_j)}{\tau}u_{j-1} + f(x,t_j,u_{j-1}) + \tau\sum\limits_{i=1}^{j-1}R_{ji}u_i}{\frac{A(x,t_j)}{\tau}}\right\|_c, \max\left\|\frac{-\frac{A(z,t_j)}{\tau}u_{ij-1} + f(x,z,t_j,u_{j-1})}{\frac{A_i(x,t_j)}{\tau}}\right\|\right\} \text{ from here}$$

$$\{\|u_j\|;\|u_{ij}\|\} \le (1 + c_2T\tau)\max\{\|u\|_j;\|u_i\|_j\} + c_1\tau .$$

By induction

$$\{\|u_j\|;\|u_{ij}\|\} \le \max\{\|\varphi(x)\|;\|\varphi_i(x,z)\|\}e^{c_2T^2} + \frac{c_1}{Tc_2}(e^{c_2T^2} - 1)$$

$c_1, c_2 -$ some constants depend on input functions.

Functions $\Phi_j = \delta_{\bar{\tau}}u_j, \ \Phi_{ij}(x,t) = \delta_{\bar{\tau}}u_{ij}$ satisfy the system of equations of type a (3), (4).

For solutions $\Phi_j(x)\ u\ \Phi_{ij}(x,t)$ by induction we uniformly estimate

$$\{\|\Phi_j\|,\|\Phi_{ij}\|\} \le c_3\exp(c_4T) + c_3\frac{\exp(c_4T) - 1}{c_4} = M_1$$

In the future, through M_k we will denote constants depending on the input data of the problems. Let us establish the uniform boundedness of the families of quantities.

$$\{\|u_j\|,\|u_{ij}\|\}, \{\|\Phi_j\|, \|\Phi_{ij}\|\}, \{\|L\Phi_j\|;\|L_z\Phi_{ij}\|\}, \{\|Lu_j\|,\|L_zu_{ij}\|\}, \{\|\delta_{\bar{\tau}}\delta_{\bar{\tau}}u_{j-12}\|;\|\delta_{\bar{\tau}}\delta_{\bar{\tau}}u_{i,j-1}\|\}$$

From the established estimates, we have

$$\left|\frac{du_j}{dx}\right| \le M_4\psi(x), \quad \left|\frac{d\Phi}{dx}\right| \le M_5\psi(x),$$

$$\begin{cases} \left|\dfrac{du_{ij}}{dz}\right| \le M_6\psi_i(z), \\ \left|\dfrac{d\Phi_{ij}}{dz}\right| \le M_7\psi_i(z) \end{cases}, \qquad z>0 \qquad (6)$$

Where $\psi(x)=\dfrac{\int_0^x m(\xi)d\xi}{K(x)}$ - uniformly bounded function. And for the function $\psi_i(x)=\dfrac{\int_0^z m_i(s)ds}{K_i(z)}$ consider the following cases:

Case 1. Limit $\overline{\lim_{z\to 0}}\psi(z)$ finite. Then the right-hand side of (6) is bounded by a constant that does not depend on the partitioning method. In the limit $\tau \to 0$ linear interpolations $u^\tau(x,t)$ u $u_i^\tau(x,z,t)$ accordingly in D and D_1 coinciding with $u(x, j\tau)$ u $u_i(x,z,j\tau)$ at $t=j\tau$ and linearly depending on t inside the layers $j\tau \le t \le (j+1)\tau$ give a solution $u(x,t)$ u $u_i(x,z,t)$, $i=1,2$ in area D u D_1 respectively

Case 2. If $\lim_{z\to +\infty}\psi_i$ $t($ is infinite or does not exist, then we cannot use estimate (6) to prove the equicontinuity of families. However, using the boundary conditions at z=0, one can be convinced of the equicontinuity of the families in this case as well.

Note 1

For the numerical solution, a modified version of the differential sweep is used [2] and is implemented by the Maple software system

Note 2

The approximate solution constructed by the method of lines converges to the exact one with the speed $o(\tau)$, $\tau-$ time step

LITERATURE

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