

RELATIONS BETWEEN CENTERS OF CIRCLES INSIDE AND OUTSIDE A TRIANGLE

Professor: Ibragimov A.M

Assistant: Pulatov O.U

Student: Haydarov I

Uzbekistan-Finland pedagogy institute

Gmail. pulatov.sertifikat@gmail.com

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Abstract:

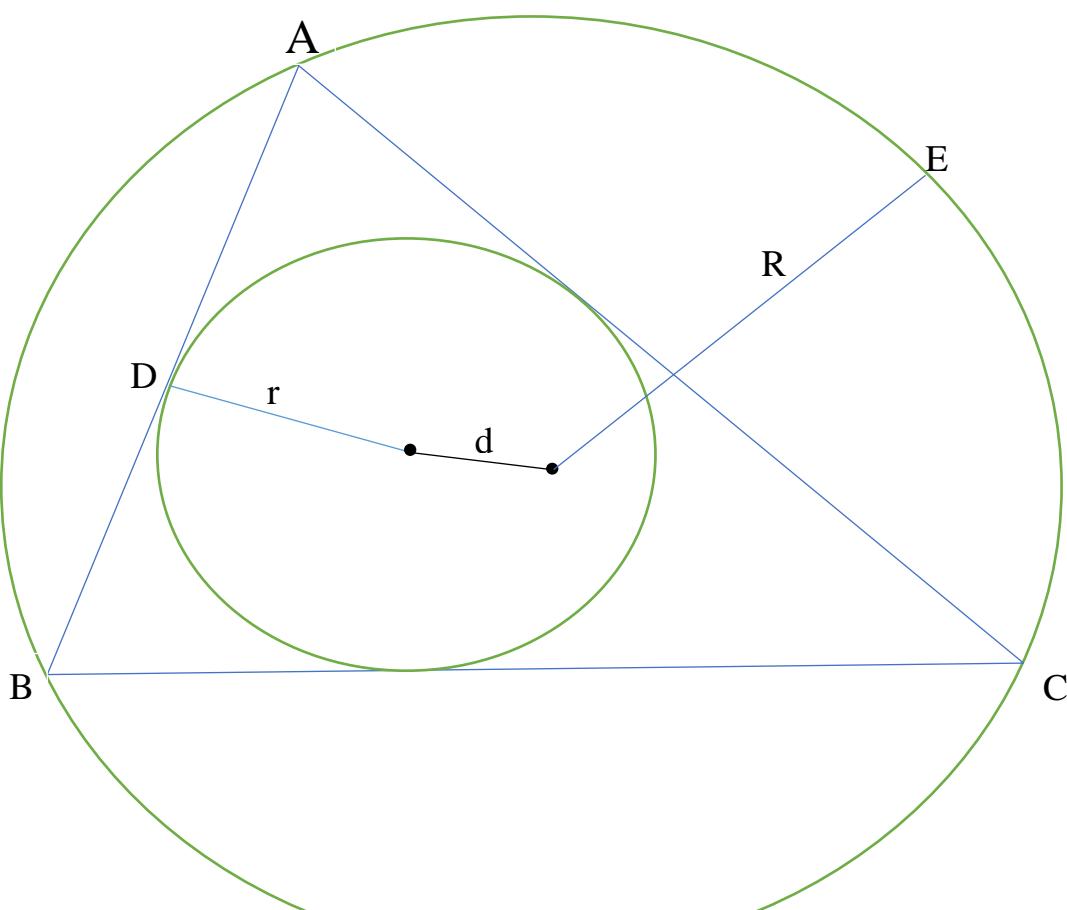
This is in the article To the triangle internal and external drawn circles centers between the distance to find about some issues in solving of students creative activities development some aspects is considered and being studied each one method analytical geometry in teaching apply according to concrete examples given

Keywords: Triangle, circles inside and outside a triangle, center, radius, angles, similarities and properties of triangles.

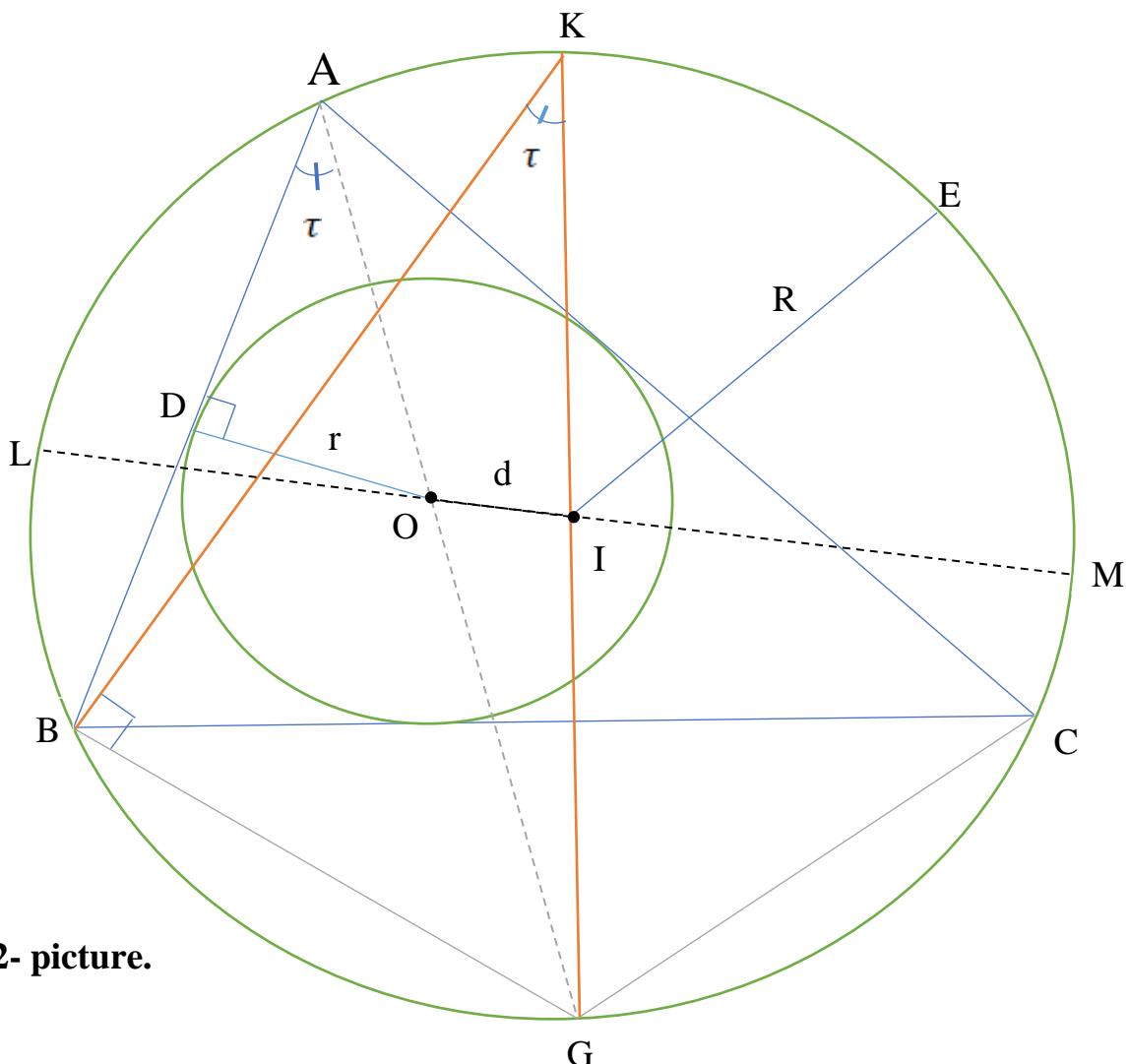
1. To the triangle internal and external drawn circles centers between the distance to find

Optional to the triangle internal and external drawn circles centers between the distance to find formula : - optional to the triangle internal drawn circle radius r to , external drawn circle radius and to R equal to if to the triangle internal and external drawn circles centers between distance to $d = \sqrt{R^2 - 2Rr}$ (1) . equal to [1]

Proof :



1- picture.



It is known to the triangle internal drawn circle center his bisectors intersection at the point will be From angle A dropped bisector and to the triangle external drawn turn around intersection Let G be the point . Then " Three _ horned about the spear to the lemma according to $|BG| = |GO| = |GC|$ equalities will be appropriate. [2]

Proof (" three horned spear lemma about "); from (Fig. 2) . it seems that $\angle BAG = \angle BKG = \beta$, , and $\angle GBC = \angle GCB$ it follows that Now |BO| we will make an incision

O point internal drawn turn around center that it was

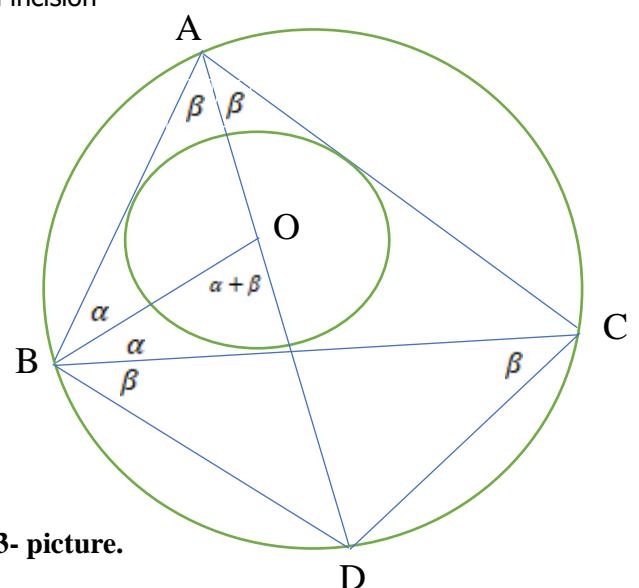
for $\angle ABO = \angle CBO = \alpha$. As a result

$$<GBO \equiv <GOB \equiv \beta + \alpha \text{ and}$$

$\leq GBC \equiv \leq GCR \equiv \beta$. From the latest results

while $|BG| \equiv |GO| \equiv |GC|$, equality is appropriate.

will be Proof finished



Now ADO and KBG similar triangles for we find the sine of the angle by $\sin \tau = \frac{r}{|AO|}$ (2) $\sin \tau = \frac{|BG|}{2R}$ (3) circular intersecting vatars property according to $|AO| \cdot |OG| = |MO| \cdot |OL|$ reasonable, from this we arrive at the following equality

$|AO| \cdot |BG| = (R - d) \cdot (R + d)$ (4). From (2) and (3) $|AO| \cdot |BG| = 2Rr$ (5).

(4) and (5) . while to $(R - d) \cdot (R + d) = 2Rr$ (6) . have we will be From (6) . If we find $d = \sqrt{R^2 + 2Rr}$ (1) expression harvest will be It has been proven . \blacktriangle

2. Solving problems for sample

Issue 1.

A , B, C of the triangle sides suitable respectively 4,5,6 ha equal to if to him internal and external drawn circles centers between find the distance .

Solution : $r = \frac{2S(ABC)}{P(ABC)}$, $R = \frac{|AB| \cdot |BC| \cdot |AC|}{4 \cdot S(ABC)}$ [3]

$$P(ABC) = (|AB| + |BC| + |AC|)$$

Of the triangle sides through

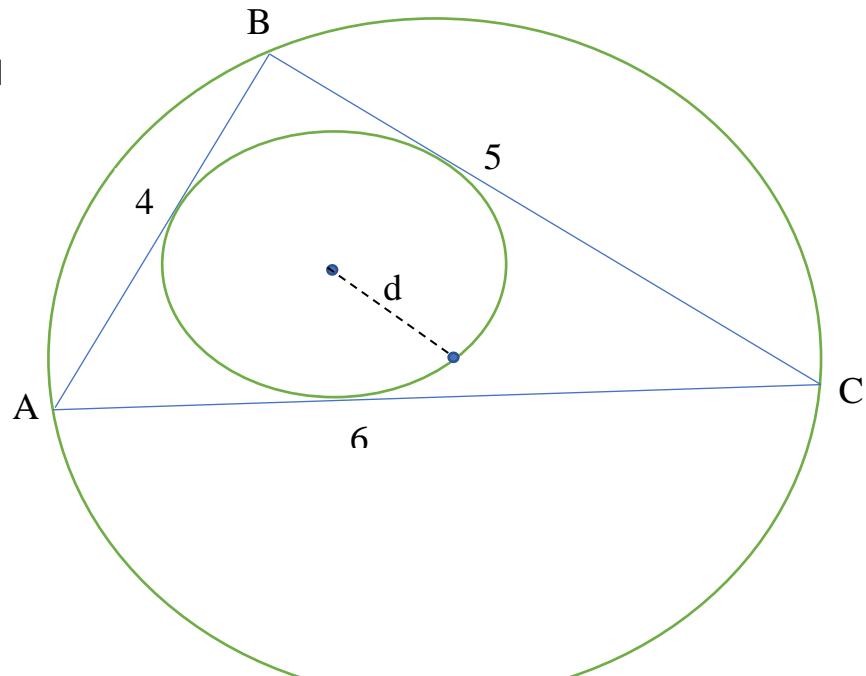
To him internal and external drawn

Circles radii find _

(1)- formula let 's put

$$r = \frac{2 \cdot S(ABC)}{P(ABC)} = \frac{\sqrt{7}}{2}$$

$$R = \frac{|AB| \cdot |BC| \cdot |AC|}{4 \cdot S(ABC)} = \frac{8}{\sqrt{7}}$$



$$d = \sqrt{R^2 - 2Rr} = \sqrt{\left(\frac{8}{\sqrt{7}}\right)^2 - 2 \cdot \left(\frac{8}{\sqrt{7}}\right) \cdot \left(\frac{\sqrt{7}}{2}\right)} = \sqrt{\frac{8}{7}}. \quad \blacksquare$$

picture 4.

Issue 2.

A circle ABCD is a rectangle internal drawn _ to the triangle ABC internal drawn from the center O to point B of the circle has been the distance is 4 ha equal to of BO continuation of the triangle ABC external drawn circle with at point D intersected into $BD = 9$ and $\angle DBC = 30^\circ$ is equal to , find the distance between the centers of the circles inscribed inside and outside ABC. [4]

Solution :

To the triangle external drawn turn around

From the center triangle ADE passing through

Let 's do it (Fig. 5). It is known that OFB triangle
the triangle DAE similar _

OFB and DAE are triangles for
of sin30 value we count .

$$\sin 30^\circ = \frac{r}{|OB|} \sin 30^\circ = \frac{|AD|}{2R}$$

$|OB| = 4$ and $|OD| = 5$ since

$$\sin 30^\circ = \frac{r}{|OB|} \Leftrightarrow \sin 30^\circ = \frac{r}{4}$$

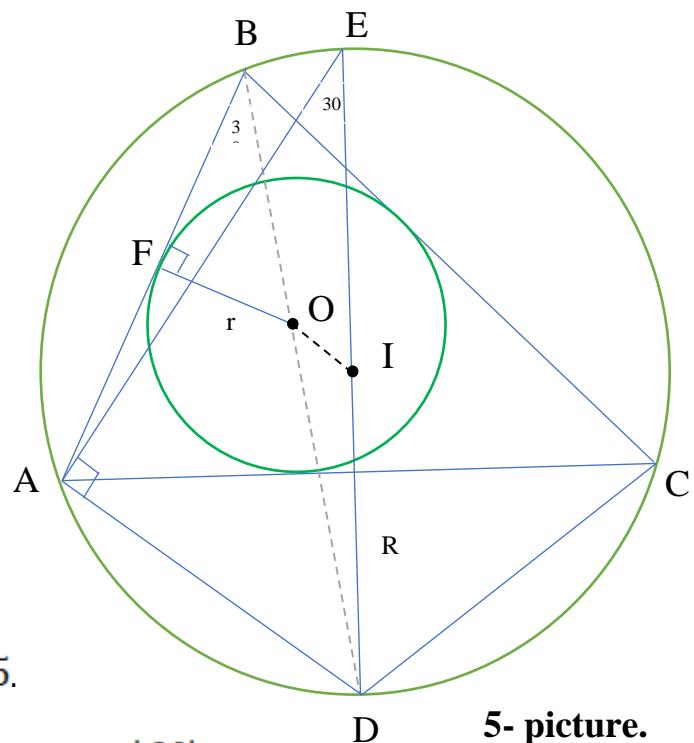
$$\Leftrightarrow r = 2.$$

$$\sin 30^\circ = \frac{|AD|}{2R} \Leftrightarrow \sin 30^\circ = \frac{5}{2R} \Leftrightarrow R = 5.$$

Now $|OI| = \sqrt{R^2 - 2Rr}$ we find the distance using the formula . $|OI|$

$$|OI| = \sqrt{5^2 - 2 \cdot 5 \cdot 2} = \sqrt{5}.$$

Answer $|OI| = \sqrt{5}.$ ■



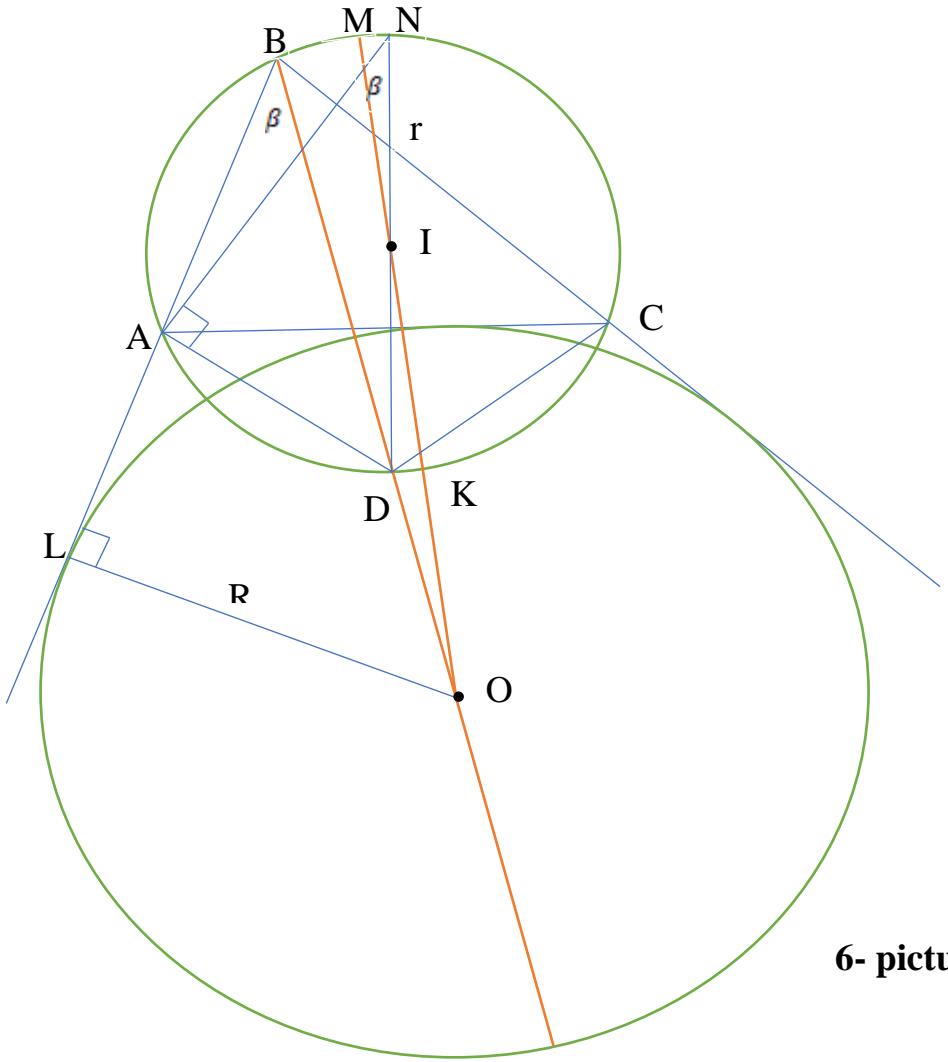
5- picture.

3. To The Triangle External And External Internal Drawn Circles Centers Between The Distance To Find

Now above sides BA and BC of the triangle continue we will deliver . As a result, to the sides internal , to the basis while external circle drawing possible and such circle to triangle external internal drawn it is called a circle .

* Optional to the triangle external and external internal drawn circles radii suitable respectively r and to R equal to let it be, then to the triangle external and external internal drawn circles centers between distance $d = \sqrt{r^2 + 2Rr}$ through the formula is considered

Proof : 



6- picture.

It is known that OLB and DAN triangles similar _ This triangles for $\sin \beta$ we calculate .

$$\sin \beta = \frac{R}{|OB|} \quad (7). \quad \sin \beta = \frac{|AD|}{2r} \quad (8).$$

(7) and (8) . equal to $|AD| \cdot |OB| = 2Rr$ We form (9).

OB and OM cutters for $|OD| \cdot |OB| = |OK| \cdot |OM|$ (10). equality appropriate .

" Three horned spear from the lemma about" the $|AD| = |DI| = |DC| = |DO|$ equality holds. (10) of the following in appearance our writing can. [8]

$$|OD| \cdot |OB| = |OK| \cdot (|OK| + 2r) \quad (11). \quad |AD| \cdot |OB| = |OD| \cdot |OB| = 2Rr \quad (12).$$

$$(11) \text{ and } (12) . \quad |OD| \cdot |OB| = |OK| \cdot (|OK| + 2r)$$

$$2Rr = |OK| \cdot (|OK| + 2r)$$

$$(|OK|)^2 + (|OK|) \cdot 2r = 2Rr \Leftrightarrow (|OK|) = \sqrt{r^2 + 2Rr} - r.$$

The distance between the centers of the circles inscribed outside and inside the triangle $|OI| = |OK| + r$ is equal to . It $|OI| = \sqrt{r^2 + 2Rr}$ follows from this.

 . It has been proven

4. Matters solve for sample _

Issue 1.

The radius is 2 ha equal to which is T -centered circumcircle of triangle ABC internal drawn is , the triangle from the end B dropped bisector circle at point D cut passes . BD 's to CD during equal to has been point I in the distance received is , from point I to AC the most short the distance is 3 ha equal to if , $|IT|$ find the distance.

solve :

the condition of the matter according to (Fig. 7) appropriate .

As follows to make done we will increase .

sides AB and BC continue which makes

If the center is I and radius to IE equal to

It was external internal circle drawing can _

(Fig. 8).

From us to the triangle radius 2 ha equal to has been to has been

External internal drawn circles centers between

The distance to find was asked was _

known to the triangle external and

external internal drawn circles

centers between $|IT|$ distance

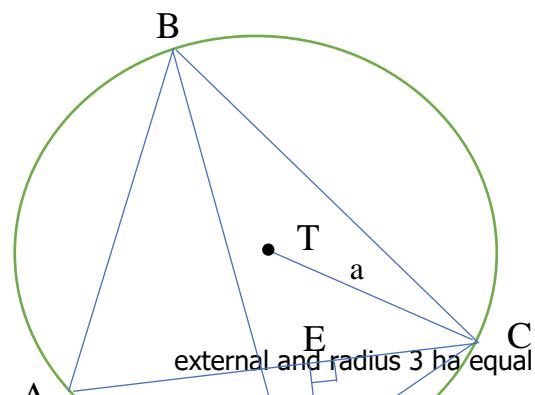
$|IT| = \sqrt{r^2 + 2Rr}$ formula

Through is determined .

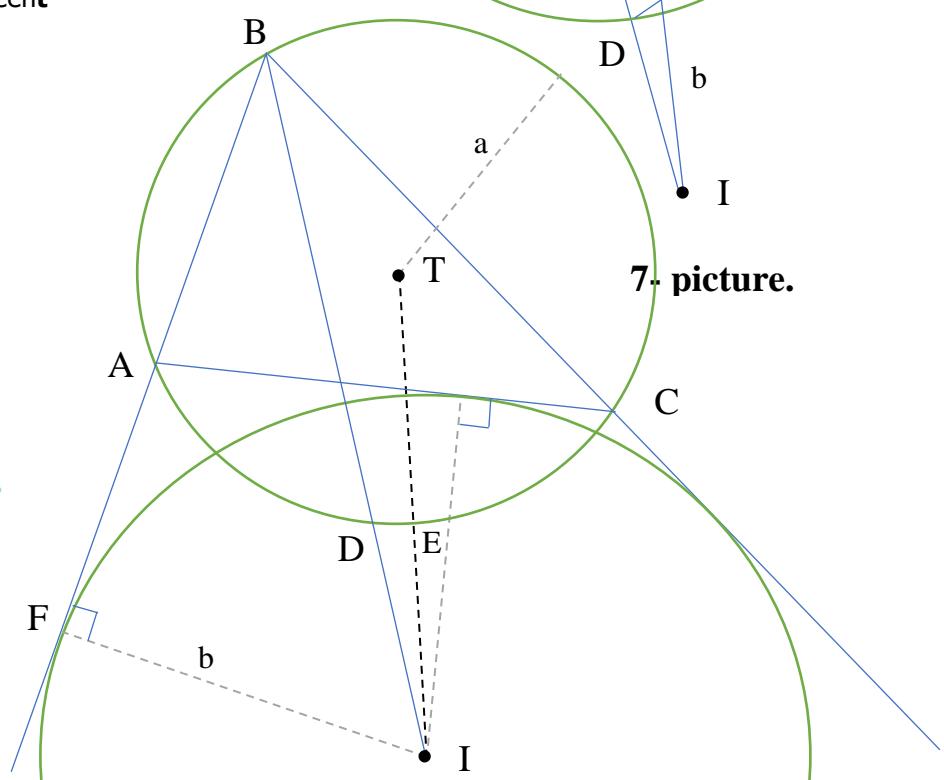
So : $r = 2, R = 3$.

Answer : $|IT| = \sqrt{2^2 + 2 \cdot 2 \cdot 3} = 4$.

. ■



7- picture.



8- picture.

Issue 2.

The radius is 12 ha equal to which is I -centered BP and BQ attempts to circle conducted . Third don't try and BP and BQ _ suitable at points M and N respectively cut passes . Harvest to the MBN form external drawn centered at O turn around radius 6 ha equal to if $|IO|$ find the distance.

Solution : □

As a result, I is centered circle to the triangle MBN external

Internal drawn circle will be

From us was asked $|IO|$ and the distance is MBN

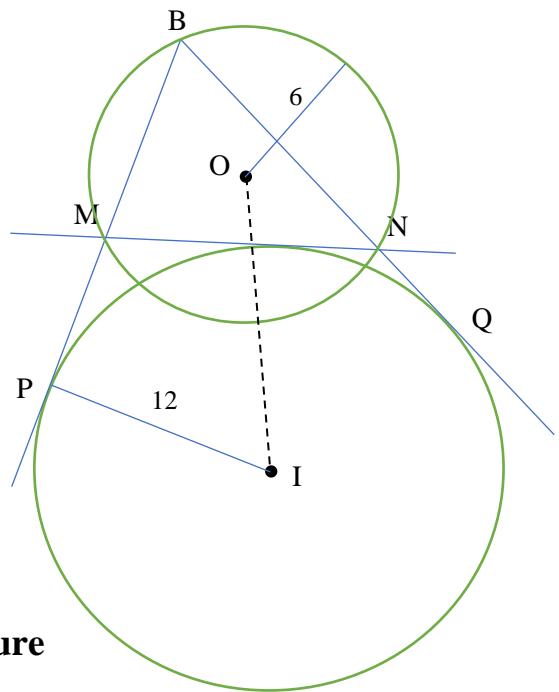
To the triangle external and external internal drawn

Circles centers between the distance gives _

$$|IO| = \sqrt{r^2 + 2rR}$$

$$\text{That is: } |IO| = \sqrt{6^2 + 2 \cdot 6 \cdot 12} = 6\sqrt{5}$$

Answer: $|IO| = 6\sqrt{5}$. ■



5. Independent solve for issues .

1 . M , N, P sides MN and NP of the triangle suitable by 10.15, $\angle DBC$ respectively $\pi - \arccos\left(\frac{1}{4}\right)$ to equal to if to him internal and external drawn circles centers between find the distance . (Answer : $10\sqrt{\frac{14}{3}}$).

2 . The circle KLM is a rectangle internal drawn _ KLM to the triangle internal drawn from the center O of the circle to the point L has been the distance is 6 ha equal to of LO continuation of the triangle KLM external drawn circle with at point N intersected into $LN = 15$ and $\angle NLM = 30^\circ$ is equal to , is internal to KLM and external drawn circles centers between find the distance . (Answer : $3\sqrt{3}$).

3 . A circle ABCD is a rectangle internal drawn _ Triangle ABC has center O radius 3 ha equal to has been circle internal drawn from point O to point B has been the distance is 6 ha equal to of BO continuation of the triangle ABC external drawn circle with at point D intersected if $BD = 14$ to ABC internal and external drawn circles centers between find the distance . (answer : 4).

4. The radius is 6 ha equal to which is T -centered circle FBN triangle internal drawn is , the triangle from the end B dropped bisector circle at point D cut passes . BD 's to ND during equal to has been point I in the distance received from point I to FN the most short distance is 9 ha equal to if , $|IT|$ find the distance. (Answer : 12) . [5]

5. The radius is 20 ha equal to is centered at P attempts BF and BE to circle conducted . Third don't try and BF and BE _ suitable at points K and T respectively cut passes . Harvest to the KBT form external drawn centered at O turn around radius 5 ha equal to if $|IO|$ find the distance . (Answer : 15) .

6. Sides are 4,6 and 6 equal to has been from the point B of the triangle dropped bisector to him external drawn circle at point D cut passes . BD 's to CD during equal to point I in the distance received and I to AC _ the most short distance is equal to the $\frac{17}{2\sqrt{2}}$ circle inscribed outside the triangle from the center to point I has been find the distance . (Answer : 3) .

REFERENCES USED.

1. A.V.Pogorelov, Analitik geometriya., T.O'qituvchi,, 1983 y.
2. Курбон Останов, Ойбек Улашевич Пулатов, Джумаев Максуд, «Обучение умениям доказать при изучении курса алгебры,» *Достижения науки и образования*, т. 2 (24), № 24, pp. 52-53, 2018.
3. Rajabov F., Nurmatov A., Analitik, geometriya va chizikli algebra, T.O'qituvchi, 1990y.

4. OU Pulatov, MM Djumayev, «In volume 11, of Eurasian Journal of Physics,,» *Development Of Students' Creative Skills in Solving Some Algebraic Problems Using Surface Formulas of Geometric Shapes*, т. 11, № 1, pp. 22-28, 2022/10/22.
5. Курбон Останов, Ойбек Улашевич Пулатов, Алижон Ахмадович Азимов, «Вопросы науки и образования,» *Использование нестандартных исследовательских задач в процессе обучения геометрии*, т. 1, № 13, pp. 120-121, 2018.
6. АА Азимзода, ОУ Пулатов, К Остонов, «Актуальные научные исследования и разработки,» МЕТОДИКА ИСПОЛЬЗОВАНИЯ СКАЛЯРНОГО ПРОИЗВЕДЕНИЯ ПРИ ИЗУЧЕНИИ МЕТРИЧЕСКИХ СООТНОШЕНИЙ ТРЕУГОЛЬНИКА, т. 1, № 3, pp. 297-300, 2017.
7. OU Pulatov, HS Aktamov, MA Muhammadiyeva, «Development of Creative Skills of Students in Solution of Some Problems of Vectoral, Mixed and Double Multiplications of Vectors,» *Eurasian Research Bulletin*, т. 14, № <https://www.geniusjournals.org/index.php/erb/article/view/2659>, pp. 224-228, 2022/11/24.
8. Джумаев М., Пулатов О. У., Остонов К. Использование сведений о дедуктивном строении математики на уроках //ББК 72 Р101. – 2017., Использование сведений о дедуктивном строении математики на уроках, г.Астана,Казахстан: Научно-издательский центр «Мир науки», 2017.
9. В.А.Погорелов , Геометрия., Ташкент: Ўқитувчи, 2001.
10. Бескин Н.М, Стереометрия, Просвещение, 1967.
11. Александров А. Д., Вернер А. Л., Рыжик В. И. Геометрия для 10–11-х классов: учеб. пособие для учащихся шк. и классов с углубленным изучением математики / А.Д. Александров, А. Л. Вернер, В. И. Рыжик – 3-е. изд. М.: , Просвещение, 1992.
12. Гайштут А., Литвиненко Г. , Стереометрия., Задачник к школьному курсу 10–11-го класса. М.: АСТ-ПРЕСС,1998. 156 с.
13. Л, С. Атанасян, В. Ф. Бутузов, С. Б. Кадомцев и др. М.: , 207 с., Геометрия: Учебник для 10–11-х кл. сред. шк./, Просвещение, 1993..
14. А. П. К. М, Киселев А. П. Элементарная геометрия. Книга для учителя 287 с., Просвещение,, 1980..
15. В. С. Крамор. М, Крамор В. С. Повторяем и систематизируем школьный курс по геометрии, Просвещение, 1992.
16. Е. С. Березанская- М , Березанская Е. С. Вопросы стереометрии в восьмилетней школе, Просвещение, 1964..
17. экстремальных задач на комбинации стереометрических фигур/, Математика в школе 1980 .