European Journal of Research Development and Sustainability (EJRDS)
Available Online at: https://www.scholarzest.com
Vol. 3 No. 4, April 2022
ISSN: 2660-5570

SOME NEW PARANORMED ZWEIER I- CONVERGENT DOUBLE SEQUENCE SPACES DESCRIBED BY DOUBLE ORLICZ FUNCTION

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Article history:
Received: 30th January 2022
Accepted: 28th February 2022
Published: 8th April 2022

Abstract:
This study introduces paranormed ideal convergent for double sequence spaces using the Zweier ideal convergent described by double Orlicz and consider some properties, such as, $2\mathfrak{H}^I(\varrho,p)$, $2\ \mathfrak{H}_0^I(\varrho,p)$, $2\Lambda^I_{\mathfrak{H}^2}\ (\varrho,p\ )$ and $2\Lambda^I_{\mathfrak{H}_0^2}(\varrho,p)$ are linear spaces and many result.

Keywords: Double Sequences, Paranormed , Ideal, Double Orlicz Function, I-Convergent

INTRODUCTION AND PRELIMINARIES

The principle of paranormed is closed related to linear metric of the spaces . That is mean very generalization of absolute values [1]. Paranormed sequence space introduced at firstly step by Nakano and Simons [2,3]. Later they examined as well by Maddox [1], Lascarides, [4,5] Tripathy and Sen [6] etc.

Sengonul defined the sequence $\mathfrak{z} = \mathfrak{z}_i$ which is often used as the Z^p alter of the sequence $n = (n_i)$ i.e,

$\mathfrak{z}_i = pn_i + (1-p)n_{i-1}$

as $n_{-1} = 0, p \neq 1, 1 < p < \infty$ and Z^p means the matrix $\mathrm{Z}^p = \mathrm{Z}_{ib_1}$ defined by

$$\mathrm{Z}_{ib_1} = \begin{cases} p, & (i = b_1) \\ 1-p, & (i-1 = b_1); i, b_1 \in \mathbb{N} \\ 0, & otherwise \end{cases}$$

[7] , therefor Basar and Altay [8], Sengonul studies the Zweier sequence spaces $\mathfrak{H}$ and $\mathfrak{H}_0$ as follows

$\mathfrak{H} = \{n = (n_{b_1}) \in \mu : \mathrm{Z}^p n \in C\}$ and

$\mathfrak{H}_0 = \{n = (n_{b_1}) \in \mu : \mathrm{Z}^p n \in C_0\}$, [7].

Kostyrko and other researchers used it to introduce the idea of I-convergence founded on the structure chart of the allowable ideal of subset for the ($\mathbb{N}$) where $\mathbb{N}$ is denoted the natural numbers ,[9]. They get more information to ideal convergence of Cakalli and Hazarika [10], Dems [11], Esi , Hazarika [12], Hazarika , Savas [13], Hazarika [14,17]. Finally it introduced by ˘Salat, Tripathy , Ziman[18]and Demirci [19] etc.

"A double Orlicz functions is a functions $\varrho : [0,\infty) \times [0,\infty) \to [0,\infty) \times [0,\infty)$such that $\varrho(n,m) = (\varrho_1(n), \varrho_2(m))$, where $\varrho_1 : [0,\infty) \to [0,\infty)$ *and* $\varrho_2 : [0,\infty) \to [0,\infty)$ such that ϱ_1, ϱ_2 are Orlicz functions which is continuous , non decreasing, even , convex , and satisfies the following conditions :

1) ϱ_1 (0) =0 , ϱ_2 (0) =0 $\Longrightarrow \varrho(0,0) = (\varrho_1(0), \varrho_2(0)) = (0,0)$
2) ϱ_1 (n) > 0 , ϱ_2 (m) > 0 $\Longrightarrow \varrho(n,m) = (\varrho_1(n), \varrho_2(m)) > (0,0)$
for $n > 0, m > 0$ *we mean by* $\varrho(n,m) > (0,0)$ *that* $\varrho_1(n) > 0$, $\varrho_2(m) > 0$
3) $\varrho_1(n) \to \infty$, $\varrho_2(m) \to \infty$ as $n,m \to \infty$ then ,
$\varrho(n,m) = (\varrho_1(n), \varrho_2(m)) \to (\infty,\infty)$ *as* $(n,m) \to (\infty,\infty)$,

They mean by $\varrho(n,m) \to (\infty,\infty)$ that $\varrho_1(n) \to \infty$, $\varrho_2(m) \to \infty$."[20].

Let $\wp$ be a non-empty set. Therefor a family of sets $I \subseteq 2^{\wp}$ (power sets of $\wp$) is said an ideal if I is additive i.e. $A, E \in I \Rightarrow A \cup E \in I$ and hereditary i.e. $A \in I, E \subseteq A \Rightarrow E \in I$,where $\wp = sup(n,m)$ [21].

Now of this article defined the paranormed Zweier I -convergent double to sequence spaces which is described by the double Orlicz functions(ϱ), $\varrho(n,m) = (\varrho_1(n), \varrho_2(m))$. Then it studied the next classes of paranormed Zweier for ideal convergent double of the sequence spaces described by the double Orlicz functions. Let C, R and $\mathbb{N}$ be the sets of all complex, real and natural numbers respectively , that set

"$\beth^2 = \{(\mathrm{n},\mathrm{m}) = (\mathrm{n}_{b_1,b_2}, \mathrm{m}_{b_1,b_2}) \ni (\mathrm{n}_{b_1,b_2}, \mathrm{m}_{b_1,b_2}) \in \Re \times \Re \text{ or } \mathbb{C} \times \mathbb{C}\}$"

Throughout this paper the double sequences shall be meant by$(\mathrm{n},\mathrm{m}) = (\mathrm{n}_{b_1,b_2}, \mathrm{m}_{b_1,b_2})$. A double infinite matrix of elements $(n_{b_1,b_2}, \mathrm{m}_{b_1,b_2})$ $\forall\, b_1, b_2 \in \mathbb{N}$, we get that

$n = (n_{b_1,b_2}), m = (m_{b_1,b_2})$ be an infinite matrix of elements $\forall\, b_1, b_2 \in \mathbb{N}$.

They used the convergence of double sequence for mean the convergence in Pringsheim"s sense in 1900, [22] . A double sequence $(n = n_{b_1,b_2})$ has a Pringsheim limit $ɟ$ (denoted by $-\text{limit}\, n = I$) provided that given $\varepsilon > 0$, there exists $y \in \mathbb{N}$ such that $|n_{b_1,b_2} - I| \le \varepsilon$ whenever $b_1, b_2 > y$. We shall describe such an n more briefly as " P-convergent" .A double sequence ($n = n_{b_1,b_2}$) is bounded if and only if there exists a positive number U such that $|n_{b_1,b_2}| < U$ for all b_1 and b_2 [23]

A paranorm is a function $ᾰ: \Gamma \to \mathbb{R}$ which satisfies the following axioms: for any $n, ʒ, n_0 \in \Gamma, \gamma, \gamma_0 \in \mathbb{C}$,

i) $ᾰ(\theta) = 0$, where θ is the zero in the complex linear space Γ

ii) $ᾰ(n) = ᾰ(-n)$,

iii) $ᾰ(n + ʒ) \le ᾰ(n) + ᾰ(ʒ)$,

iv) The scalar multiplication is continuous, that is $\gamma \to \gamma_0$, $n \to n_0$ implies $\gamma n \to \gamma_0\, n_0$ .In other words, $|\gamma - \gamma_0| \to 0$, $ᾰ(n - n_0) \to 0$ imply $ᾰ(\gamma n - \gamma_0\, ᾰ_0) \to 0$. A paranormed space is a linear space Γ with a paranorm $ᾰ$ and it is written as $(\Gamma, ᾰ)$.

Any function $ᾰ$ which satisfies all the conditions (i)-(iv) together with the condition (v) $ᾰ(n) = 0$ if only if $n = \theta$ is called a total paranorm on Γ and the pair $(\Gamma, ᾰ)$ is called total paranormed space . [24]

Lemma . A sequence space B is solid implies that B is monotone .

Remark . If ϱ is an Orlicz function, then $\varrho(\lambda n) \le \lambda \varrho(n)$ for all λ with $0 < \lambda < 1$.

2. MAIN RESULTS

An Zweier ideal convergent studying the following classes of double sequence spaces.

$(\mathfrak{H}^2)^I = \{b_1, b_2 \in \mathbb{N} : \{(n,m) = (n_{b_1,b_2}, m_{b_1,b_2}) \in ℶ^2 :$
I $-\lim(Ƶ^2)^p(n,m) = (\mathfrak{f}_1, \mathfrak{f}_2)$ $for\ some$ $\mathfrak{f}_1, \mathfrak{f}_2\}\} \in I$, where
I $-\lim(Ƶ^2)^p n = \mathfrak{f}_1$ $for\ some$ $\mathfrak{f}_1$, I $-\lim(Ƶ^2)^p m = \mathfrak{f}_2$ $for\ some$ $\mathfrak{f}_2$

$(\mathfrak{H}_0^2)^I = \{b_1, b_2 \in \mathbb{N} : \{(n,m) = (n_{b_1,b_2}, m_{b_1,b_2}) \in ℶ^2 : I -\lim(Ƶ^2)^p(n,m) = (0,0)\} \in I$, where
I $-\lim(Ƶ^2)^p n = 0$, I $-\lim(Ƶ^2)^p m = 0$

$(\mathfrak{H}_\infty^2)^I = \{b_1, b_2 \in \mathbb{N} : \{(n,m) = (n_{b_1,b_2}, m_{b_1,b_2}) \in b_1, b_2 : sup_{b_1,b_2}|(Ƶ^2)^p\ (n,m)| < (\infty,\infty)\}\} \in I$, where
$sup_{b_1,b_2}|(Ƶ^2)^p\ n| < \infty$, $sup_{b_1,b_2}|(Ƶ^2)^p\ m| < \infty$.

We also denote by

$\left(\Lambda^2_{\mathfrak{H}^2}\right)^I = (\mathfrak{H}_\infty^2)^I \cap (\mathfrak{H}^2)^I$ and $\left(\Lambda^2_{\mathfrak{H}_0^2}\right)^I = (\mathfrak{H}_\infty^2)^I \cap (\mathfrak{H}_0^2)^I$

And now we introduce the classes of paranormed Zweier I -Convergent double sequence spaces defined by the double Orlicz functions.

$$2\mathfrak{H}^I(\varrho, p) = \{(n,m) = (n_{b_1,b_2}, m_{b_1,b_2}) : \left\{(v_1,v_2) \in N \times N : \left[\varrho_1\left(\frac{\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1 v_2} - \mathfrak{f}_1}{\rho}\right), \varrho_2\left(\frac{\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1 v_2} - \mathfrak{f}_2}{\rho}\right)\right]^{p_{v_1 v_2}} \ge \varepsilon\right\} \in I\},$$

$for\ some$ $\mathfrak{f}_1, \mathfrak{f}_2 \in \mathbb{C}$ and $\rho > 0$.

$$2\ \mathfrak{H}_0^I\ (\varrho, p) = \{(n,m) = (n_{b_1,b_2}, m_{b_1,b_2}) : \left\{(v_1,v_2) \in N \times N : \left[\varrho_1\left(\frac{\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1 v_2}}{\rho}\right), \varrho_2\left(\frac{\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1 v_2}}{\rho}\right)\right]^{p_{v_1 v_2}} \ge \varepsilon\right\} \in I\},$$

$for\ some$ $\mathfrak{f}_1, \mathfrak{f}_2 \in \mathbb{C}$ and $\rho > 0$.

$$2\ \mathfrak{H}_\infty^I\ (\varrho, p) = \{(n,m) = (n_{b_1,b_2}, m_{b_1,b_2}) : \left\{(v_1,v_2) \in N \times N : \exists k > 0 \left[\varrho_1\left(\frac{\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1 v_2}}{\rho}\right), \varrho_2\left(\frac{\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1 v_2}}{\rho}\right)\right]^{p_{v_1 v_2}} \ge k\right\} \in I\},$$

$for\ some$ $\mathfrak{f}_1, \mathfrak{f}_2 \in \mathbb{C}$ and $\rho > 0$.

Also we denoted by

$2\Lambda^I_{\mathfrak{H}^2}(\varrho, p) = 2\mathfrak{H}_\infty^I(\varrho, p) \cap 2\mathfrak{H}^I(\varrho, p)$ and $2\Lambda^I_{\mathfrak{H}_0^2}(\varrho, p) = 2\mathfrak{H}_\infty^I(\varrho, p) \cap 2\mathfrak{H}_0^I(\varrho, p)$.

We will denote by $(Ƶ^2)^p(n_{b_1,b_2}, m_{b_1,b_2}) = (n'_{b_1,b_2}, m'_{b_1,b_2})$,where $(Ƶ^2)^p(n_{b_1,b_2}) = (n'_{b_1,b_2})$ and $(Ƶ^2)^p(m_{b_1,b_2}) = (m'_{b_1,b_2})$, $(Ƶ^2)^p(ʆ_{b_1,b_2}, ʋ_{b_1,b_2}) = (ʆ_{b_1,b_2}, ʋ'_{b_1,b_2})$, where $(Ƶ^2)^p(ʆ_{b_1,b_2}) = (ʆ_{b_1,b_2})$ and $(Ƶ^2)^p(ʋ_{b_1,b_2}) = (ʋ'_{b_1,b_2})$.

where $p = (p_{v_1 v_2})$is a double sequence of positive real numbers

Theorem 1 .

For any double Orlicz function ϱ, the classes of double sequences $2\mathfrak{H}^I(\varrho, p)$, $2\,\mathfrak{H}_0^I(\varrho, p)$, $2\Lambda^{\,I}_{\mathfrak{H}^2}(\varrho, p\,)$ and $2\Lambda^{I}_{\mathfrak{H}_0^2}(\varrho, p)$ are linear spaces.

Proof .

We will prove the result for the space $2\mathfrak{H}^I(\varrho, p)$.The proof for the other spaces will follow similarly.

Let $n = (\,n_{b_1,b_2}), m = (\,m_{b_1,b_2}) \in 2\mathfrak{H}^I(\varrho, p)$ and let (α, α) , (β, β) be scalars. Then there exists positive numbers ρ_1 and ρ_2 such that

$$A_1 = \left\{(n, m) = (\,n_{b_1,b_2}, m_{b_1,b_2}) : \left\{(v_1, v_2) \in N \times N : \left[\varrho_1\left(\frac{\left|\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_1}\right), \varrho_2\left(\frac{\left|\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_1}\right)\right]^{p_{v_1v_2}} \geq \frac{\varepsilon}{2}\right\} \in I\right\},$$

1 for some $\mathfrak{f}_1 \in \mathbb{C}$

$$A_2 = \left\{(\mathfrak{z}, \mathfrak{Y}) = (\,\mathfrak{z}_{b_1,b_2}, \mathfrak{Y}_{b_1,b_2}) : \left\{(v_1, v_2) \in N \times N : \left[\varrho_1\left(\frac{\left|\left((Ƶ^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_2}\right), \varrho_2\left(\frac{\left|\left((Ƶ^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_2}\right)\right]^{p_{v_1v_2}} \geq \frac{\varepsilon}{2}\right\} \in I\right\} \quad ,$$

2 for some $\mathfrak{f}_2 \in \mathbb{C}$

That is for a given $\varepsilon > 0$, we have

Let ρ_0=max $\{2|\,\alpha\,|\rho_1, 2|\,\beta\,|\,\rho_2\}$. Since ϱ_1, ϱ_2 *and* ϱ are non – decreasing and convex functions ,we have

$$\left[\varrho_1\left(\frac{\left|\left(\alpha\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1v_2} + \beta\left((Ƶ^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right) - (\alpha\mathfrak{f}_1 - \beta\mathfrak{f}_2)\right|}{\rho_0}\right)^{p_{v_1v_2}}, \varrho_2\left(\frac{\left|\left(\alpha\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1v_2} + \beta\left((Ƶ^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right) - (\right.}{\rho_0}\right.\right.$$

$$\leq \left[\left[\frac{|\alpha|\left|\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_0}\right]^{p_{v_1v_2}} + \left[\frac{|\beta|\left|\left((Ƶ^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_0}\right]^{p_{v_1v_2}}, \left[\frac{|\alpha|\left|\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_0}\right]^{p_{v_1v_2}}\right.$$

$$\left. + \left[\frac{|\beta|\left|\left((Ƶ^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_0}\right]^{p_{v_1v_2}}\right]$$

$$\leq \left[\left[\frac{\left|\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_1}\right]^{p_{v_1v_2}} + \left[\frac{\left|\left((Ƶ^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_1}\right]^{p_{v_1v_2}}, \left[\frac{\left|\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_2}\right]^{p_{v_1v_2}}\right.$$

$$\left. + \left[\frac{\left|\left((Ƶ^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_2}\right]^{p_{v_1v_2}}\right]$$

$$\left\{(v_1, v_2) \in N \times\right.$$

$$N : \left[\varrho_1\left(\frac{\left|\alpha\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1v_2} + \beta\left((Ƶ^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right) - (\alpha\mathfrak{f}_1 - \beta\mathfrak{f}_2)\right|}{\rho_0}\right)^{p_{v_1v_2}}, \varrho_2\left(\frac{\left|\alpha\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1v_2} + \beta\left((Ƶ^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right) - (\alpha\mathfrak{f}_1 - \beta\mathfrak{f}_2)\right|}{\rho_0}\right)^{p_{v_1v_2}}\right] \geq$$

$$\left.\varepsilon\right\} \subseteq \left\{(v_1, v_2) \in N \times N : \left[\varrho_1\left(\frac{\left|\left((Ƶ^2)^p(n_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_1}\right), \varrho_2\left(\frac{\left|\left((Ƶ^2)^p(m_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_1\right|}{\rho_1}\right)\right]^{p_{v_1v_2}} \geq \frac{\varepsilon}{2}\right\} \cup \left\{(v_1, v_2) \in N \times\right.$$

$$\left. N : \left[\varrho_1\left(\frac{\left|\left((Ƶ^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_2}\right), \varrho_2\left(\frac{\left|\left((Ƶ^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2} - \mathfrak{f}_2\right|}{\rho_2}\right)\right]^{p_{v_1v_2}}\right\} \subseteq A_1 \cup A_2 \in I$$

Now

$$\left[\alpha\left(\left((Ƶ^2)^p(n_{b_1,b_2}, \mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right) + \beta\left(\left((Ƶ^2)^p(m_{b_1,b_2}, \mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right)\right] \in 2\mathfrak{H}^I(\varrho, p)$$

Then $2\mathfrak{H}^I(\varrho, p)$ is a linear space. ■

Theorem.2

The spaces $2\Lambda^{\,I}_{\mathfrak{H}^2}(\varrho, p\,)$ and $2\,\Lambda^{I}_{\mathfrak{H}_0^2}(\varrho, p)$ are paranormed spaces, with the paranorm $\breve{a}(n, m)$ defined by

$$ǎ(n,m) = inf\left\{\rho^{\frac{p_{v_1v_2}}{Ч}} : sup_{v_1v_2}\left|\varrho_1\left(\frac{\left|\left((Z^2)^p(n_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(m_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho}\right)\right| \leq 1, for\ some\ \rho > 0\right\}$$

where $Ч = \{I, sup_{v_1v_2}p_{v_1v_2}\}$.

Proof.
Clearly $ǎ(-n,-m) = ǎ(n,m) and\ ǎ(0,0) = (0,0)$
Let $(n,m) = (n_{b_1,b_2}, m_{b_1,b_2})\ and\ (\mathfrak{z},\mathfrak{Y}) = (\mathfrak{z}_{b_1,b_2}, \mathfrak{Y}_{b_1,b_2})$ be two elements in $2\Lambda^{I}_{\mathfrak{H}^2}(\varrho, p)$ where
$\rho_1, \rho_2 > 0$

$$A_1 = \left\{\rho_1 : sup_{v_1v_2}\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(n_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(m_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right)\right] \leq 1\right\}$$

And

$$A_2 = \left\{\rho_2 : sup_{v_1v_2}\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right)\right] \leq 1\right\}$$

We take $\rho = \rho_1 + \rho_2$. Be using the convexity of double orlicz functions , we get

$$\left[\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(n_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right) + \varrho_1\left(\frac{\left|\left((Z^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right)\right], \left[\varrho_2\left(\frac{\left|\left((Z^2)^p(m_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right) + \varrho_2\left(\frac{\left|\left((Z^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right)\right]\right] \leq$$

$$\frac{\rho_1}{\rho_1+\rho_2}\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(n_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(m_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right)\right] + \frac{\rho_2}{\rho_1+\rho_2}\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right)\right]$$

Which in terms give us,

$$\left[sup_{v_1v_2}\left[\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(n_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right) + \varrho_1\left(\frac{\left|\left((Z^2)^p(\mathfrak{z}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right)\right], \left[\varrho_2\left(\frac{\left|\left((Z^2)^p(m_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_1}\right) + \varrho_2\left(\frac{\left|\left((Z^2)^p(\mathfrak{Y}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_2}\right)\right]\right]\right]^{p_{v_1v_2}} \leq 1$$

And

$$ǎ[(n,m) + (\mathfrak{z},\mathfrak{Y})] = inf\left\{(\rho_1+\rho_2)^{\frac{p_{v_1v_2}}{Ч}} : \rho_1 \in A_1, \rho_2 \in A_2\right\} \leq inf\left\{\rho_1^{\frac{p_{v_1v_2}}{Ч}} : \rho_1 \in A_1\right\} + inf\left\{\rho_2^{\frac{p_{v_1v_2}}{Ч}} : \rho_2 \in A_2\right\}$$
$$= ǎ(n,m) + ǎ(\mathfrak{z},\mathfrak{Y})$$

let $k^{\Lambda} \to \mathfrak{f}$, where $k^{\Lambda}, \mathfrak{f} \in \mathbb{C}$ and let $ǎ[(n^{\Lambda}, m^{\Lambda}) - (n,m)] \to 0$ as $\Lambda \to \infty$.
Now prove $ǎ(k^{\Lambda}(n^{\Lambda}, m^{\Lambda}) - \mathfrak{f}(n,m)) \to 0$ as $\Lambda \to \infty$,and put

$$A_3 = \left\{\rho_{\Lambda} > 0 : sup_{v_1v_2}\left[\left[\varrho_1\left(\frac{\left|\left((Z^2)^p({n^{\Lambda}}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_{\Lambda}}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p({m^{\Lambda}}_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_{\Lambda}}\right)\right]\right]^{p_{v_1v_2}} \leq 1\right\}$$

And

$$A_4 = \left\{\rho_{ь} > 0 : sup_{v_1v_2}\left[\left[\varrho_1\left(\frac{\left|\left((Z^2)^p({n^{\Lambda}}_{b_1,b_2} - n_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_{ь}}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p({m^{\Lambda}}_{b_1,b_2} - m_{b_1,b_2})\right)_{v_1v_2}\right|}{\rho_{ь}}\right)\right]\right]^{p_{v_1v_2}} \leq 1\right\}$$

by the continuity ϱ ,we obtain that

$$\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(k^{\Lambda}{n^{\Lambda}}_{b_1,b_2} - \mathfrak{f}n_{b_1,b_2})\right)_{v_1v_2}\right|}{|k^{\Lambda} - \mathfrak{f}|\rho_{\Lambda} + |\mathfrak{f}|\rho_{ь}}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(k^{\Lambda}{m^{\Lambda}}_{b_1,b_2} - \mathfrak{f}m_{b_1,b_2})\right)_{v_1v_2}\right|}{|k^{\Lambda} - \mathfrak{f}|\rho_{\Lambda} + |\mathfrak{f}|\rho_{ь}}\right)\right] \leq$$

$$\left[\varrho_1\left(\frac{\left|\left((Z^2)^p(k^{\Lambda}{n^{\Lambda}}_{b_1,b_2} - \mathfrak{f}{n^{\Lambda}}_{b_1,b_2})\right)_{v_1v_2}\right|}{|k^{\Lambda} - \mathfrak{f}|\rho_{\Lambda} + |\mathfrak{f}|\rho_{ь}}\right), \varrho_2\left(\frac{\left|\left((Z^2)^p(k^{\Lambda}{m^{\Lambda}}_{b_1,b_2} - \mathfrak{f}{m^{\Lambda}}_{b_1,b_2})\right)_{v_1v_2}\right|}{|k^{\Lambda} - \mathfrak{f}|\rho_{\Lambda} + |\mathfrak{f}|\rho_{ь}}\right)\right] +$$

$$\left[\varrho_1\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(\text{ƒ}n^{\wedge}{}_{b_1,b_2}-\text{ƒ}n_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}}\right),\varrho_2\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(\text{ƒ}m^{\wedge}{}_{b_1,b_2}-\text{ƒ}m_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}}\right)\right]\le$$

$$\frac{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}}{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}}\left[\varrho_1\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(n^{\wedge}{}_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{\rho_{\wedge}}\right),\varrho_2\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(m^{\wedge}{}_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{\rho_{\wedge}}\right)\right]+$$

$$\frac{|\text{ƒ}|\rho_{ь}}{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}}\left[\varrho_1\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(n^{\wedge}{}_{b_1,b_2}-n_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{\rho_{ь}}\right),\varrho_2\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(m^{\wedge}{}_{b_1,b_2}-m_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{\rho_{ь}}\right)\right].$$

Of that inequality it follows the

$$sup_{v_1v_2}\left[\varrho_1\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(k^{\wedge}n^{\wedge}{}_{b_1,b_2}-\text{ƒ}n_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}}\right),\varrho_2\left(\frac{\left|\left((\text{Ƶ}^2)^p\left(k^{\wedge}m^{\wedge}{}_{b_1,b_2}-\text{ƒ}m_{b_1,b_2}\right)\right)_{v_1v_2}\right|}{|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}}\right)\right]^{p_{v_1v_2}}\le 1$$

Then

$$\text{ᾰ}\left(k^{\wedge}(n^{\wedge}{}_{b_1,b_2},m^{\wedge}{}_{b_1,b_2})-\text{ƒ}(n_{b_1,b_2},m_{b_1,b_2})\right)=inf\left\{[|k^{\wedge}-\text{ƒ}|\rho_{\wedge}+|\text{ƒ}|\rho_{ь}]^{\frac{p_{v_1v_2}}{\text{ч}}}:\rho_{\wedge}\in A_3,\rho_{ь}\in A_4\right\}$$
$$\le|k^{\wedge}-\text{ƒ}|^{\frac{p_{v_1v_2}}{\text{ч}}}inf\left\{(\rho_{\wedge})^{\frac{p_{v_1v_2}}{\text{ч}}}:\rho_{\wedge}\in A_3\right\}+|\text{ƒ}|^{\frac{p_{v_1v_2}}{\text{ч}}}\left\{(\rho_{ь})^{\frac{p_{v_1v_2}}{\text{ч}}}:\rho_{ь}\in A_4\right\}\le max\left\{1,|k^{\wedge}-\text{ƒ}|^{\frac{p_{v_1v_2}}{\text{ч}}}\right\}\text{ᾰ}(n^{\wedge}{}_{b_1,b_2},m^{\wedge}{}_{b_1,b_2})+$$
$$max\left\{1,|\text{ƒ}|^{\frac{p_{v_1v_2}}{\text{ч}}}\right\}\text{ᾰ}\left[(n^{\wedge}{}_{b_1,b_2},m^{\wedge}{}_{b_1,b_2})-(n_{b_1,b_2},m_{b_1,b_2})\right]\qquad(1)$$

Then

$$\text{ᾰ}(n^{\wedge}{}_{b_1,b_2},m^{\wedge}{}_{b_1,b_2})\le\text{ᾰ}(n_{b_1,b_2},m_{b_1,b_2})+\text{ᾰ}\left[(n^{\wedge}{}_{b_1,b_2},m^{\wedge}{}_{b_1,b_2})-(n_{b_1,b_2},m_{b_1,b_2})\right]\ for\ all\ \wedge\in\mathbb{N}$$

now, the right hand side is used by option of the relation (1) tends to 0 as $\wedge\to\infty$ and the prove that. ■

Corollary

$\mathcal{M}\subseteq\mathcal{M}[\varrho(n,m),p], where\ \mathcal{M}=2\mathfrak{H}^I, 2\ \mathfrak{H}_0^I,\left(\wedge_{\mathfrak{H}^2}^2\right)^I$ and $\left(\wedge_{\mathfrak{H}_0^2}^2\right)^I$.

Theorem .3

Let $p=(p_{v_1v_2})$ and $x=(x_{v_1v_2})$ be two double sequences of positive real numbers. Then $\left(\wedge_{\mathfrak{H}_0^2}^2\right)^I[\varrho(n,m),p]\supseteq\left(\wedge_{\mathfrak{H}_0^2}^2\right)^I[\varrho(n,m),x]$ if and only if $\lim_{v_1v_2\in K}inf\frac{p_{v_1v_2}}{x_{v_1v_2}}>0, where\ K\subseteq\mathbb{N}\ such\ that\ K\in F(I)$.

Proof.

Let $\lim_{v_1v_2\in K}inf\frac{p_{v_1v_2}}{x_{v_1v_2}}>0\ and\ (n_{v_1,v_2},m_{v_1,v_2})\ \in\left(\wedge_{\mathfrak{H}_0^2}^2\right)^I[\varrho(n,m),x]$.

Then there exists $\delta>0\ such\ that\quad p_{v_1v_2}>\delta_{x_{v_1v_2}}$, for all $v_1,v_2\in K$.

Let $(n_{b_1,b_2},m_{b_1,b_2})\ \in\left(\wedge_{\mathfrak{H}_0^2}^2\right)^I[\varrho(n,m),x]\ then$ for given $\varepsilon>0$, we have

$$\text{E}=\left\{(n,m)=\left(n_{b_1,b_2},m_{b_1,b_2}\right):\left\{(v_1,v_2)\in N\times N:\left[\varrho_1\left(\frac{\left((\text{Ƶ}^2)^p(n_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right),\varrho_2\left(\frac{\left((\text{Ƶ}^2)^p(m_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right)\right]^{p_{v_1v_2}}\ge\varepsilon\right\}\in I\ \right\}.$$

Let $B_0=E\cup K^c$. Therefor we get $B_0\in I$. Now we get $v_1,v_2\in B_0$,

$$\left\{(n,m)=\left(n_{b_1,b_2},m_{b_1,b_2}\right)\ :\left\{(v_1,v_2)\in N\times N:\left[\varrho_1\left(\frac{\left((\text{Ƶ}^2)^p(n_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right),\varrho_2\left(\frac{\left((\text{Ƶ}^2)^p(m_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right)\right]^{p_{v_1v_2}}\ge\varepsilon\right\}\in I\right\}\subseteq\{(n,m)=$$

$$\left(n_{b_1,b_2},m_{b_1,b_2}\right):\left\{(v_1,v_2)\in N\times N:\left[\varrho_1\left(\frac{\left((\text{Ƶ}^2)^p(n_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right),\varrho_2\left(\frac{\left((\text{Ƶ}^2)^p(m_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right)\right]^{\delta_{x_{v_1v_2}}}\ge\varepsilon\right\}\in I\ \}$$

Then we get

$$\left\{(n,m)=\left(n_{b_1,b_2},m_{b_1,b_2}\right):\left\{(v_1,v_2)\in N\times N:\left[\varrho_1\left(\frac{\left((\text{Ƶ}^2)^p(n_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right),\varrho_2\left(\frac{\left((\text{Ƶ}^2)^p(m_{b_1,b_2})\right)_{v_1v_2}}{\rho}\right)\right]^{x_{v_1v_2}}\ge\varepsilon\right\}\in I\right\}$$

$$(n_{b_1,b_2},m_{b_1,b_2})\in\left(\wedge_{\mathfrak{H}_0^2}^2\right)^I[\varrho(n,m),p]$$

And the converse part of the proof follows obviously.■

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