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BIERMAN-SCHWINGER PRINCIPLE FOR A SINGLE MODEL PARTIAL-INTEGRAL OPERATOR

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Abstract:
The article deals with a model operator consisting of a set of two special integral operators. Important spectra of the model operator K. The Biermann-Schwinger principle is proved for the operator K, i.e. the number of eigenvalues to the left of the number of the operator K is equal to the number of eigenvalues greater than 1 for some compact operators. $\sigma_{ess}(K)$ $\lambda<\inf\sigma_{ess}(K)\ T(\lambda)$

Keywords: integral operators, Biermann Schwinger principle

Let be a Cartesian product, a Hilbert space of quadratically integrable functions on . Suppose that a single measure is chosen, i.e. $T^{\nu}, T=(-\pi,\pi], (T^{\nu})^n=\underbrace{T^{\nu}\times T^{\nu}\times\ldots\times T^{\nu}}_{n}\ L_2(T^{\nu})^n-(T^{\nu})^n\ T^{\nu}$

$$\int_{T^{\nu}} dp=1.$$

Consider the operator in where the operators and act in by formulas $K=K_1+K_2\ L_2((T^{\nu})^2),\ K_1\ K_2$ $L_2((T^{\nu})^2)$

$$(K_1 f)(p,q)=\int_{T^{\nu}} K_1(p,q,t)\,f(t,q)dt,$$

$$(K_2 f)(p,q)=\int_{T^{\nu}} K_2(p,q,t)\,f(p,t)dt.$$

Here, nuclei and $K_1(p,q,t)=K_1(t,q,p)\ K_2(p,q,t)=K_2(p,t,q)$ real-significant continuous functions on $(T^{\nu})^3$.

Lemma 1. The operator K operating in Hilbert space is linear, limited, and self-conjugated. $L_2((T^{\nu})^2)$

Proof of Lemma 1. Let be arbitrary elements. Then from the linearity of the integral we have $\alpha,\beta\in C$ $f,g\in L_2\left((T^{\nu})^2\right)$

$$\left(K_1(\alpha f+\beta g)\right)(p,q)=\int_{T^{\nu}} K_1(p,q,t)(\alpha f(t,q)+\beta g(t,q))dt=$$

$$=\alpha\int_{T^{\nu}} K_1(p,q,t)\,f(t,q)dt+\beta\int_{T^{\nu}} K_1(p,q,t)g(t,q)dt=$$

$$=\alpha\left(K_1 f\right)(p,q)+\beta\left(K_1 g\right)(p,q).$$

Similarly, the linearity of the operator . An operator as a sum of linear operators is a linear operator. K_2 K

Now let's show the limitation of the operator . Denote by . Let , then K_1 $M = \max_{p,q,t\in T^\nu} |K_1(p,q,t)|$ $f \in L_2\left(\left(T^\nu\right)^2\right)$

$$\|K_1 f\|^2 = \iint\left|\int K_1(p,q,t) f(t,q)dt\right|^2 dpdq.$$

In the latter equality, using Hölder's inequality, we have

$$\|K_1 f\|^2 \le \iint \left[\int |K_1(p,q,t)|^2 dt \int |f(t,q)|^2 dt\right] dpdq \le M^2 \|f\|^2.$$

From here for anyone we get $f \in L_2\left(\left(T^\nu\right)^2\right)$

$$\|K_1 f| \le |M\|f\|.$$

Similarly, the limitation of the operator . The amount of limited operators is a limited operator, therefore limited. K_2 K

To prove the self-adhesion of the operator, it is enough to show equality K
$(Kf,g)=(f,Kg)$, for any $f,g \in L_2\left(\left(T^\nu\right)^2\right)$.

First we will prove the self-adhesion of the operator . Let be arbitrary elements. Then K_1 $f,g \in L_2\left(\left(T^\nu\right)^2\right)$

$$(K_1 f,g) = \iint \left\{\int K_1(p,q,t) f(t,q)dt\right\} \overline{g(p,q)} dpdq =$$
$$= \iint f(t,q) \left\{\int K_1(p,q,t) \overline{g(p,q)} dp\right\} dtdq.$$

By replacing the variables and in the integral on the right side of the last equality, we get $t=p$ $p=s$

$$(K_1 f,g) = \iint f(t,q) \left\{\int \overline{\overline{K_1(p,q,t) g(s,q)}} ds\right\} dpdq.$$

Hence, by virtue of the condition, we have $K_1(p,q,t) = \overline{K_1(t,q,p)},$

$$(K_1 f,g) = \iint f(p,q) \left\{\int \overline{K_1(p,q,t) g(t,q)} dt\right\} dpdq = (f, K_1, g).$$

Thus, the self-conjugation of the operator . Similarly, the self-conjugation of the operator is proved. The sum of self-adjoint operators is a self-adjoint operator. Therefore, the operator is also self-adjoint. K_1 K_2 $K = K_1 + K_2$

Lemma 1. Proven.

Denote by the spectrum of the operator . $\sigma(A)$ A

The article considers the model operator K consisting of the sum of two partial-integral operators and finds an essential spectrum of this operator $\sigma_{ess}(K)$

Let the operator act in space according to the formula $T(z)$ $L_2((T^\nu)^2)$

$$(T(z)f)(p,q) = I - (R_1(z)R_2(z)f)(p,q),$$

where is the resolvent of the operator . $R_i(z) = (K_i - zI)^{-1}, i=1,2$ $K_i, i=1,2$

For operator K is the case.

Lemma 2. *Число* $z \in C \setminus (\sigma(K_1) \cup \sigma(K_2))$ *is the eigenvalue of the K operator if and only if it is the eigenvalue of the operator* $\lambda = 1$ $T(z)$.

Using lemma 1 and Fredholm's analytic theorem, applying methods similar to those in [1] and [2,3],we get.

Theorem 1. *For the spectrum of the operator K there is an equality* $\sigma(K)$ $\sigma_{ess}(K) = \sigma(K_1) \cup \sigma(K_2)$.

Note that the spectra and operators, respectively, and in detail studied in the work [2]. $\sigma(K_1)$ $\sigma(K_2)$ K_1 K_2

Let A be a self-conjugated bounded operator operating in Hilbert space and whose subspace of space whose elements satisfy the condition H $\mathrm{H}_A(\lambda),\ \lambda > \sup \sigma_{ess}(A),\ H\ (Af, f) > \lambda(f, f),\ f \neq 0.$

Put

$$n(\lambda, A) = \sup_{H_A(\lambda)} \dim H_A(\lambda).$$

The number coincides with the number of eigenvalues (taking into account the multiplicity) of the operator A lying to the right of . Let's denote through the positive square root of the operator of the positive operator A. $n(\lambda, A)$ λ $A^{1/2}$

Lemma 3. *For each, operators* $z < \inf \sigma_{ess}(K)$ $K_1 - zI$ $K_2 - zI$ *are both positive operators and there is equality.*

$$(R_2(z))^{\frac{1}{2}} K_1 (R_2(z))^{\frac{1}{2}} = \frac{1}{z} K_1 + K_{12}(z), \qquad (1)$$

where each is a compact operator. $K_{12}(z)$ $z < \inf \sigma_{ess}(K)$

Proof. it $z < \inf \sigma_{ess}(K)$ *follows that* $z < \inf \sigma(K_i),\ i = 1,2.$ therefore , i.e. operators and are positive for all $zI - K_i < 0,\ i = 1,2$ $K_1 - zI$ $K_2 - zI$ $z < \inf \sigma_{ess}(K)$.

Noticing the existence of an operator everywhere in Hilbert space Hence the representation $\inf \sigma_{ess}(K) \le 0$ $(R_2(z))^{\frac{1}{2}} + \sqrt{-z} I)^{-1}$ $L_2((T^{\nu})^2).$

$$(R_2(z))^{\frac{1}{2}} - \frac{1}{\sqrt{-z}} I = \left((R_2(z))^{\frac{1}{2}} + \frac{1}{\sqrt{-z}} I \right)^{-1} \left((R_2(z))^{\frac{1}{2}} + \frac{1}{\sqrt{-z}} I \right) \left((R_2(z))^{\frac{1}{2}} - \frac{1}{\sqrt{-z}} I \right) =$$
$$= \left((R_2(z))^{\frac{1}{2}} + \frac{1}{\sqrt{-z}} I \right)^{-1} \left(R_2(z)^{\frac{1}{2}} - \frac{1}{-z} I \right). \qquad (2)$$

It is easy to verify that the operator is a compact operator [2]. Therefore, according to (2) we have the compactness of the operator for all . From here we can easily get (1). $(R_2(z) - \frac{1}{-z} I) K_1$ $z < \inf \sigma_{ess}(K)$ $((R_2(z))^{\frac{1}{2}} - \frac{1}{-z} I) K_1$ $z < \inf \sigma_{ess}(K)$

Note that the Biermann-Schwinger principle plays an important role in determining the finiteness or infinity of the discrete spectrum of the operator in question[4,5].

Here is the Biermann-Schwinger principle for the operator K, which we will prove by the same method as work [2,3]

Theorem 2. *For each, equality is performed* $z < \inf K$

$$n(z, K) = n\left(1, -\left(\left[\frac{1}{z} K_1 - I \right]^{-1} \right)^{\frac{1}{2}} K_{12}(z) \left(\left[\frac{1}{z} K_1 - I \right]^{-1} \right)^{\frac{1}{2}} \right).$$

Proof. First we will prove that

$$n(z, K) = n\left(1, -(R_2(z))^{\frac{1}{2}} K_1 (R_2(z))^{\frac{1}{2}} \right).$$

Suppose that i.e. $u \in H_k(-z),\ (Ku, u) < z(u, u).$

$$((K_2 - zI)u,u) < -(K_1 u,u).$$

Therefore

$$(y,y) < -(R_2(z))^{\frac{1}{2}} K_1 (R_2(z))^{\frac{1}{2}} y, y), \quad y = (K_2 - zI)^{\frac{1}{2}} u.$$

Thus $n(z,K) \le n(1, -(R_2(z))^{\frac{1}{2}} K_1 (R_2(z))^{\frac{1}{2}}.$

Reasoning similarly, we get the opposite statement

$$n(z,K) \ge n(1, -(R_2(z))^{\frac{1}{2}} K_1 (R_2(z))^{\frac{1}{2}}.$$

At the same time, according to the formula (1)

$$n(z,K) = n(1, \frac{1}{z} K_1 + K_{12}(z)).$$

Again applying the variational principle, we have

$$n(1, \frac{1}{z} K_1 + K_{12}(z)) = n\left(1, -\left(\left[\frac{1}{z} K_1 - I\right]^{-1}\right)^{\frac{1}{2}} K_{12}(z) \left(\left[\frac{1}{z} K_1 - I\right]^{-1}\right)^{\frac{1}{2}}\right).$$

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