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NON-TRADITIONAL METHODS OF INTEGRATING RATIONAL FUNCTIONS

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Article history:		Abstract:
Received:	October 10th 2021	The article discusses various non-traditional methods of integrating rational functions. Using these methods, the integration of correct and incorrect fractions as well as rational functions is reduced to the integration of simple expressions. Achieving the ultimate whole is simplified
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This article is scientific and methodological in nature and offers users some methods for calculating integrals. Briefly consider the method of unknown coefficients of integration of rational functions. It is known that the ratio of two algebraic polynomials, namely $f(x)=\dfrac{P_m(x)}{Q_n(x)}$ (1)

the expression is called a rational function or rational fraction. Here

$P_m(x)=b_0+b_1x+b_2x^2+b_3x^3+\cdots+b_mx^m$ va

$Q_n(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n$ ($a_n, b_m \neq 0; m\geq 0, n\geq 1$) are considered polynomials with real coefficients.

If $m<n$, then $\dfrac{P_m(x)}{Q_n(x)}$ correct rational function, when $m\geq n$ the improper fraction is called a rational function [2].

Note 1. If (1) the rational fraction is an irregular fraction, then the speed of the fraction is divided by the denominator,

$$f(x)=w(x)+\frac{P_k(x)}{Q_n(x)},(k<n) \tag{2}$$

reduced to the form, here $w(x)$ a lot.

As is known from higher algebra, any $Q_n(x)$ polynomial is such

$$Q_n(x)=a_n(x-\alpha)(x-\beta)\cdots(x-\gamma) \tag{3}$$

(here $a_n-Q_n(x)$ coefficient in front of the highest level x in the polynomial, $\alpha,\beta,\cdots,\gamma$ appear $Q_n(x)=0$ roots of the equation) can be described as.

If the roots of the polynomial are equal to each other, then the polynomial is

$$Q_n(x)=a_n(x-\alpha)^r(x-\beta)^s\cdots(x-\gamma)^t \tag{4}$$

ko'rinishga keltiriladi, bu yerda $r,s,\cdots,t$ natural sonlar, $\alpha,\beta,\cdots,\gamma$ lar mos ravishda $Q_n(x)$ko'phadning $r,s,\cdots,t$ karrali haqiqiy ildizlari deyiladi va $r+s+\cdots+t\ =n$ bo'ladi. [2, 33-bet]

Ko'phadning (3) dagi ildizlari ichida kompleks ildizlar ham bo'lishi mumkin.

It is known from the algebra course that if $\alpha=a+bi$ is an r-fold root of a polynomial with real coefficients, then the sum $\overline{\alpha}=a-bi$ is also an r-fold root of the polynomial. In other words, if (4) contains $(x-\alpha)^r,(\alpha=a+bi)$ then (4) also contains $(x-\overline{\alpha})^r,(\overline{\alpha}=a-bi)$ and so on

$$(x-\alpha)^r(x-\overline{\alpha})^r=\{[x-(a+bi)]\cdot[x-(a-bi)]\}^r=$$
$$\left[x^2-(a+bi)x-(a-bi)x+a^2+b^2\right]^r=\left[x^2-2ax+a^2+b^2\right]^r=\left(x^2+2px+q\right)^r$$

where $p=-a, q=a^2+b^2, p^2-q<0,$, p end q are real numbers. In addition, if we follow the above process for other complex roots, then (4) will look like this:

$$Q_n(x)=A_n(x-\alpha)^r(x-\beta)^s\cdots\left(x^2+2ux+v\right)^k\cdot(x+2px+q)^t\cdots \tag{5}$$

where $r,s,\cdots,k,t...$- is a natural number, and $\alpha,\beta,\cdots,p,q,u,v$ is a real number.

In the course of algebra, the following theorem was proved, but not proved.

Theorem 1. If $\frac{P_m(x)}{Q_n(x)}$ is represented by the polynomial $Q_n(x)$ (5) in a direct rational fraction, then the rational fraction is the only form

$$\frac{P_m(x)}{Q_n(x)}=\frac{A_1}{x-\alpha}+\frac{A_2}{(x-\alpha)^2}+...+\frac{A_r}{(x-\alpha)^r}+...+\frac{M_1x+N_1}{\left(x^2+2px+q\right)}+$$
$$+\frac{M_2x+N_2}{\left(x^2+2px+q\right)^2}+...+\frac{M_tx+N_t}{\left(x^2+2px+q\right)^t} \tag{6}$$

It is designated as, where $A_1,A_2,\cdots A_r,\cdots M_1,N_1,M_2,N_2,\cdots M_t,N_t$ are unknown real numbers. To find the unknown coefficients in (6), the same levels of $P_m(x)$ in the polynomial x with the polynomial formed on the right-hand side by the theorem on the equality of two polynomials by reducing (6) to a common denominator. As a result of the equation of the front coefficients, a system of linear algebraic equations for the unknown coefficients is formed. From this system, we find the unknown coefficients and put the found values into (6). This method of finding unknown coefficients in the distribution of fractions is called the method of unknown coefficients [1], [2].

Note 2. When integrating regular rational fractions, linear algebra must solve a system of equations. But solving a system of linear algebraic equations is not always easy. In such cases (special cases), it is necessary to use methods that are more convenient than the method of unknown coefficients. For example, Hewiside's method, value selection method, Horner's scheme, methods of using differentiation[2].

Heaviside method.

If $\frac{P_m(x)}{Q_n(x)},(m<n)$ is a fraction of a direct fraction

$$Q_n(x)=a_n(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n),a_n=1 \tag{7}$$

if so $\alpha_1,\alpha_2,\cdots,\alpha_n$ - here are the real numbers , $Q_n(\alpha_i)=0; i=\overline{1,n}.$ It is expedient to find the distribution coefficients in simple fractions of the form (6) by the Hewiside method. Let's implement this method as follows.

Step 1. The denominator of the direct fraction $\frac{P_m(x)}{Q_n(x)},(m<n)$ is written through the factors of the expression $Q_n(x)$ (7), ie

$$\frac{P_m(x)}{Q_n(x)}=\frac{P_m(x)}{(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)}$$

we can write an equation.

Step 2. By temporarily closing the $Q_n(x)$ from $(x-\alpha_i),\left(i=\overline{1,n}\right)$ we replace, i with x in α_i in factors that do not close at every Yu value.. This gives the number α_i for each A_i, $i=\overline{1,n}$ root:

$$A_1 = \frac{P_m(\alpha_1)}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\cdots(\alpha_1-\alpha_n)},$$

$$A_2 = \frac{P_m(\alpha_2)}{(\alpha_2-\alpha_1)(\alpha_2-\alpha_3)\cdots(\alpha_2-\alpha_n)},$$

....

$$A_n = \frac{P_m(\alpha_n)}{(\alpha_n-\alpha_1)(\alpha_n-\alpha_2)\cdots(\alpha_n-\alpha_{n-1})}.$$

Step 3. $\frac{P_m(x)}{Q_n(x)}, (m<n)$ rational fraction

$$\frac{P_n(x)}{Q_n(x)} = \frac{A_1}{x-\alpha_1} + \frac{A_2}{x-\alpha_2} + \cdots + \frac{A_n}{x-\alpha_n}$$

then we integrate the values $A_1, A_n, \cdots, A_n$ and arrive at the desired result [2].

Example 1. This

$$J = \int \frac{3x^4+3x^3-13x^2+4}{x^5-5x^3+4x}dx$$

considered whole.

Solve.

Step 1. It's like $P_4(x) = 3x^4+3x^3-13x^2+4$, $Q_5(x) = x^5-5x^3+4x = x(x^4-5x^2+4) = x(x-1)(x+1)(x-2)(x+2)$ here. As a result

$$\frac{P_4(x)}{Q_5(x)} = \frac{3x^4+3x^3-13x^2+4}{x(x-1)(x+1)(x-2)(x+2)}, \ \alpha_1 = 0, \alpha_2 = 1, \alpha_3 = -1, \alpha_4 = 2, \alpha_5 = -2$$

we will have.

Step 2.

$\alpha_1 = 0$ da $A = \frac{4}{(-1)\cdot 1\cdot(-2)\cdot 2} = 1,$

$\alpha_2 = 1$ da $B = \frac{-3}{1\cdot 2\cdot(-1)\cdot 3} = \frac{1}{2},$

$\alpha_3 = -1$ da $C = \frac{3-3-13+4}{(-1)(-2)\cdot(-3)\cdot 1} = \frac{3}{2},$

$\alpha_4 = 2$ da $D = \frac{3\cdot 16+3\cdot 8-13\cdot 4+4}{2\cdot 1\cdot 3\cdot 4} = 1,$

$\alpha_5 = -2$ da $E = \frac{3\cdot 16-3\cdot 8-13\cdot 4+4}{(-2)\cdot(-3)\cdot(-1)\cdot(-4)} = -1$

values. For the given integral

$$J = \int \frac{3x^4+3x^3-13x^2+4}{x(x-1)(x+1)(x-2)(x+2)}dx = \int \frac{A}{x}dx + \int \frac{B}{x-1}dx + \int \frac{C}{x+1}dx + \int \frac{D}{x-2}dx + \int \frac{E}{x+2}dx =$$

$$= \int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x-1}dx + \frac{3}{2}\int \frac{1}{x+1}dx + \int \frac{1}{x-2}dx - \int \frac{1}{x+2}dx = \ln|x| + \frac{1}{2}\ln|x-1| + \frac{3}{2}\ln|x+1| +$$ we come to the

$$+\ln|x-2| - \ln|x+2| + C$$

relationship [2].

How to select values.
In many cases, it is also convenient to find unknown coefficients by selecting the values of x, for example, $x = 0, \pm 1, \pm 2, \ldots$.

Example 2. This

$$J = \int \frac{x^2+1}{(x-1)(x-2)(x-3)} dx$$

be considered integral.

Solve. Let's look at finding the unknowns $\frac{P_4(x)}{Q_5(x)} = \frac{x^2+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ in equation A, B, C In the above equation, the total denominator is $x^2+1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ Put the values $x=1, x=2, x=3$ in a row in the above equation:

$$1+1 = A\cdot(-1)\cdot(-2) \Leftrightarrow 2 = 2\cdot A \Leftrightarrow A = 1$$

$$2^2+1 = -B \Leftrightarrow B = -5$$

$$3^2+1 = 2C, C = 5$$

We will have values. As a result

$$J = \int \frac{x^2+1}{(x-1)(x-2)(x-3)} dx = \int \frac{A}{x-1} dx + \int \frac{B}{x-2} dx + \int \frac{C}{x-3} dx = \int \frac{1}{x-1} dx - 5\int \frac{dx}{x-2} +$$

$$+5\int \frac{dx}{x-3} = \ln|x-1| - 5\ln|x-2| + 5\ln|x-3| + C$$

we conclude [2].

How to use differentiation.

This method works best when the $\frac{P_m(x)}{Q_n(x)}, (m<n)$ straight fraction has $Q_n(x)$ denominators, with real multiple roots.

Example 3. This

$$J = \int \frac{x-1}{(x+2)^3} dx$$

be considered integral.

Solve. $\frac{P_1(x)}{Q_3(x)} = \frac{x-1}{(x+2)^3}$ under integral for direct rational function

$$\frac{x-1}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

we have a spread of A, B, C unknown coefficients in this equation

Let's look at the process of finding by the method of differentiation.

Step 1. We get rid of the fraction by bringing the right side of the above equation to the common denominator

$$x-1 = A(x+2)^2 + B(x+2) + C$$

If we say $x=-2$ in this equation, we get $C=-3$.

Step 2. If we differentiate (ie derive) both sides of the above equation with respect to x, then

$$1 = 2A(x+2) + B$$

we have equality. In this equation, $x=-2$ equals $B=1$.

Step 3. If we differentiate both sides of the last equation with respect to x, the result

$$2A = 0, A = 0$$

we come to equality. So we get $A=0, B=1, C=-3$ substituting these values into the above distribution, we obtain the following for a given integral, i.e.

$$J=\int\frac{x-1}{(x+2)^3}dx=\int\frac{1}{(x+2)^2}dx-3\int\frac{1}{(x+2)^3}dx=-\frac{1}{x+2}+\frac{3}{2}\cdot\frac{1}{(x+2)^2}+C$$

we conclude [2].

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